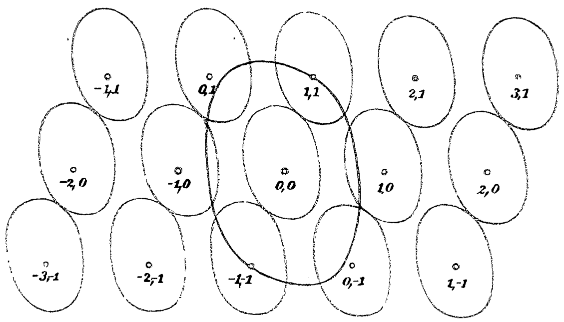
Zur Geometrie der Zahlen.

Fig. 1. Zahlengitter und konvexe Kurven.



$f(x, y)$:

(1) $f(x, y) > 0,\ \ x, y \neq 0,0;\ \ f(0,0) = 0,$

(2) $f(tx, ty) = t f(x, y),\ \ t > 0,$

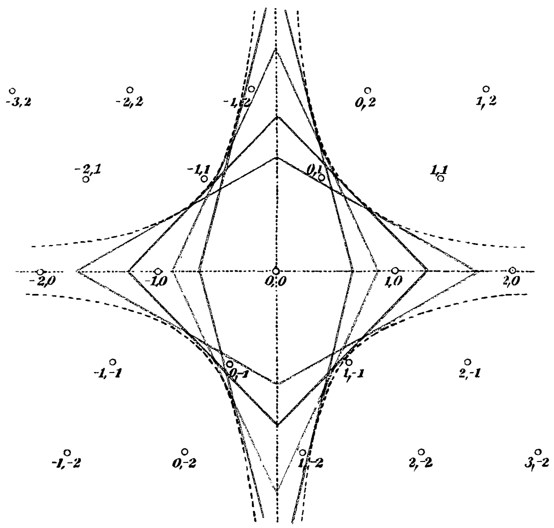
(3) $f(-x, -y) = f(x, y),$

(4) $f(x_1, y_1) + f(x_2, y_2) \geq f(x_1 + x_2, y_1 + y_2),$

(5) $f(x, y) \leq 1,\ \ \iint dx\,dy = J;$

(6) $f(x, y) \leq \frac{2}{\sqrt{J}}.$

Fig. 2. Diagonalketten.



(1) $f(z) = c_m z^m + \cdots + c_0 + \frac{c_1}{z} + \frac{c_2}{z^2} + \cdots$

$= F_0(z) - \cfrac{1}{F_1(z) - \cfrac{1}{F_2(z) - \cdots}};$

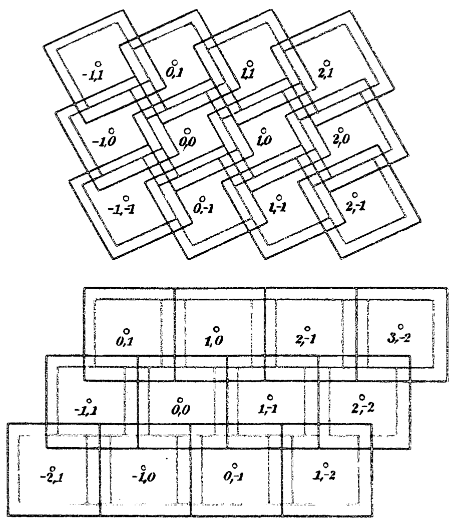
(2) $\big(P(z) - f(z)\,Q(z)\big)\,Q(z).$

(3) $\left|\frac{x}{y} - a\right| < \frac{1}{2y^2},$ (4) $a = g_0 - \cfrac{1}{g_1 - \cfrac{1}{g_2 - \cdots}}$

(5) $\xi = \alpha x + \beta y,\ \ \eta = \gamma x + \delta y,\ \ \alpha\delta - \beta\gamma = 1;$

(6) $-\frac{1}{2} < \xi\eta < \frac{1}{2}.$

Fig. 3. Inhomogene lineare Ausdrücke.

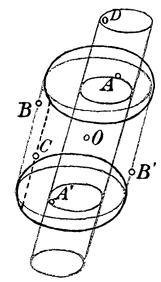


(1) $|x - ay - b| < \frac{1}{4|y|}.$

(2) $\xi = \alpha x + \beta y,\ \ \eta = \gamma x + \delta y,$ $\alpha\delta - \beta\gamma = 1;$

(3) $|(\xi - \xi_0)(\eta - \eta_0)| < \frac{1}{4}.$

Fig. 4. Kettenalgorithmen für drei lineare Formen.



1° Eine reelle, zwei konjugiert komplexe Formen:

$$\begin{cases} \xi = \alpha x + \beta y + \gamma z, \\ \eta = (\alpha' + i\alpha'')x + (\beta' + i\beta'')y + (\gamma' + i\gamma'')z, \\ \zeta = (\alpha' - i\alpha'')x + (\beta' - i\beta'')y + (\gamma' - i\gamma'')z; \end{cases}$$

(1) $-\lambda \leq \xi \leq \lambda,\ \ |\eta| \leq \mu.$

2° Drei reelle Formen:

$\xi = \alpha x + \beta y + \gamma z,$

$\eta = \alpha' x + \beta' y + \gamma' z,$ Det. $\neq 0;$

$\zeta = \alpha'' x + \beta'' y + \gamma'' z,$

(2) $|\xi|,\ |\eta|,\ |\zeta| \leq \sqrt[3]{|\mathrm{Det.}|}.$

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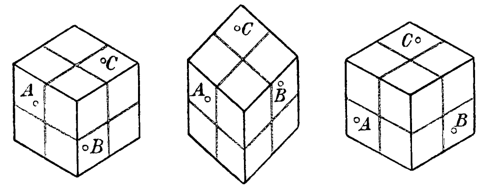
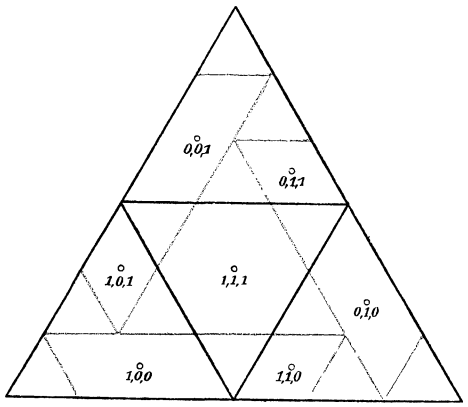


$A + B + C$ nicht im Inneren des Parallelepipeds; $A + B$ u. $A + B - C$ nicht im Inneren; $A + B + C = 0.$

Fig. 5. Dichteste Lagerung von Oktaedern.



$\varphi = -\xi + \eta + \zeta,\ \ \chi = \xi - \eta + \zeta,\ \ \psi = \xi + \eta - \zeta,$
$\omega = \xi + \eta + \zeta,$

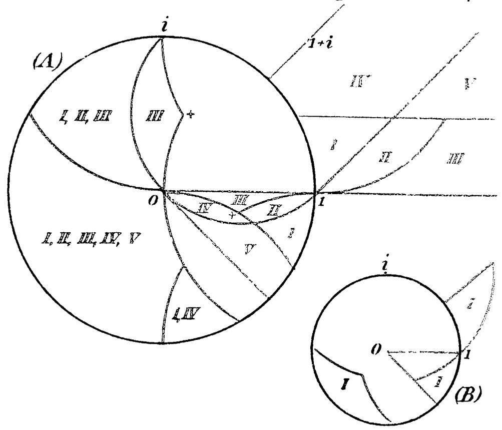
(1) $\varphi + \chi + \psi + \omega = 0,$ Det. $(\xi, \eta, \zeta) = \Delta;$

(2) $|\varphi|,\ |\chi|,\ |\psi|,\ |\omega| \leq \sqrt[3]{\frac{108}{19}\Delta}.$

(3) $\pm(x - az) \pm \frac{z}{t} = 1,\ \ \pm(y - bz) \pm \frac{z}{t} = 1;$

(4) $\left|\frac{x}{z} - a\right|,\ \left|\frac{y}{z} - b\right| < \sqrt{\frac{8}{19}}\,\frac{1}{z^{\frac{3}{2}}},\ \left(\sqrt{\frac{8}{19}} = 0{,}648\ldots\right)$

Fig. 6. Lineare Formen im Körper von $i = \sqrt[4]{1}$.

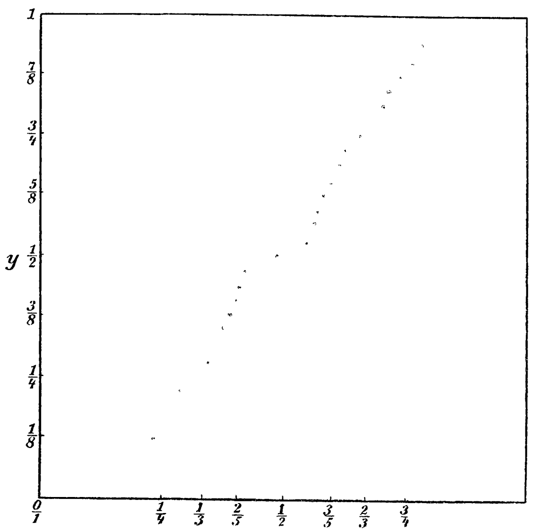


(1) $\xi = (\alpha + i\alpha')(x + ix') + (\beta + i\beta')(y + iy'),$
$\eta = (\gamma + i\gamma')(x + ix') + (\delta + i\delta')(y + iy'),$ Det. $(\xi, \eta) = \Delta;$

(2) $\begin{vmatrix} x_1 + ix_1', & x_2 + ix_2' \\ y_1 + iy_1', & y_2 + iy_2' \end{vmatrix},$ (3) $\xi = \lambda e^{i\varphi}\big(X + iX' + \varrho(Y + iY')\big),$
$\eta = \mu e^{i\psi}\big(\sigma(X + iX') + Y + iY'\big),$

(4) $|\xi|,\ |\eta| \leq \sqrt{\frac{\sqrt{3} + 1}{\sqrt{6}}\,|\Delta|}.$

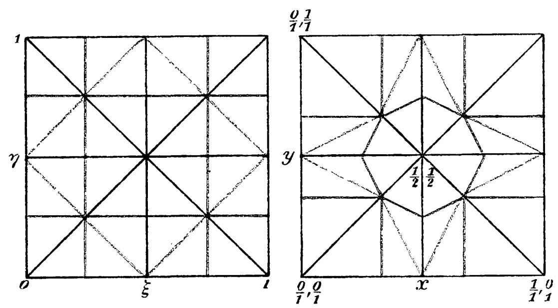
Fig. 7.
Kriterium für die reellen quadratischen Irrationalzahlen.



$\frac{a}{b} \cdots \frac{a + a'}{b + b'} \cdots \frac{a'}{b'}.$

$y = ?(x)$: x quadratische Irrationalzahl, y rational und nicht dyadisch;
x rational, y dyadisch.

Fig. 8.
Kriterium für die reellen kubischen Irrationalzahlen.



$\frac{a}{c},\ \frac{b}{c} \cdots \frac{a + a'}{c + c'},\ \frac{b + b'}{c + c'} \cdots \frac{a'}{c'},\ \frac{b'}{c'},$

(1) $\xi = \varphi(x, y),\ \ \eta = \psi(x, y).$

1, x, y unabhängige Zahlen in einem kubischen Körper;
ξ, η rational,
von den Zahlen ξ, η, $\xi - \eta$, $\xi + \eta$ keine dyadisch.