

The balance

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Aristotle steelyard, Mechanical problems, 20.853b–854a, 1849b–850a, 848a–850b10

Heron OO II

Vitruvius 10,3,4

Isidore, Orig. 16,25,6

Hero, *Mechanica* 2,7

Pappus *Collectio* 8,2; 1068:20

Plutarch, *Vita Marcelli*, xiv.7

Singer, vol. 1, pp. 779–84.

Heron, *Dioptra* 308.19–20, simple machines, 310.26, 312.20. Heronis iii

Pappus, *Mathematical Collection* 8.52 [?, p. 49]:

There are five machines by the use of which a given weight is moved by a given force, and we will undertake to give the forms, the applications, and the names of these machines. Now both Hero and Philo have shown that these machines, though they are considerably different from one another in their external form, are reducible to a single principle. The names are as follows: wheel and axle, lever, system of pulleys, wedge, and, finally, the so-called endless screw.

Philon, *Belopoeica* 59.11–15 [?, p. 123]:

Larger circles have a bigger turning effect than smaller ones fixed about the same centre, as we demonstrated in our ‘Principles of Leverage’; similarly, men move loads with levers more easily when they place the fulcrum as close as possible to the load (the fulcrum, of course, performs the function of the centre; when brought close to the load it decreases the circle and easy movement is the result).

Pappus, *Mathematical Collection* 8.1–2 [?, pp. 47–48]:

The science of mechanics, my son Hermodorus, is useful for many applications in daily life. In addition, it is found worthy of great favour among the philosophers and is eagerly pursued by all mathematicians. For it has very nearly primary relevance to the science of nature, which concerns the material of the elements making up the universe. Being the general, theoretical consideration of inertia and mass and of movement through space, it not only examines the causes of objects moved according to their nature, but also causes objects to leave their own position against opposing forces, contrary to their nature, devising it through the theories suggested to it by matter itself. The mechanicians (*mechanikoi*) of Hero's school state that part of the science of mechanics is theoretical, part is practical. The theoretical part consists of geometry, arithmetic, astronomy, and physics, while the practical part consists of metal-working, construction, carpentry, painting, and the manual practice of these arts. They say, then, that he will be the best inventor and master builder of mechanical contrivances who from his boyhood has been involved with the fields of knowledge mentioned above and has received training in the above-mentioned crafts, and having a nature inclined towards them. But since it is not possible for the same person to grasp so many fields of knowledge and at the same time learn the above-mentioned crafts, they advise the individual who desires to undertake projects involving mechanics to use the particular crafts of which he himself has a command for the purposes suitable to each.

Of all the mechanical arts the most indispensable with respect to human needs are the following (the mechanical considered before the architectural): the art of the “conjurers” (*manganarioi*), termed mechanicians (*mechanikoi*) by the ancients. By means of machines (*mechanai*) they lift great weights to a height, moving them with little force, contrary to nature. Also, the art of those who construct the siege machines necessary for war (*organopoioi*), they too are termed mechanicians. They design catapults that fire stone or iron missiles and other objects of this type a great distance. Additional to these is the art of those properly called machine-builders (*mechanopoioi*). Water is quite easily raised from a great depth

by means of the water-lifting machines which they devise. The ancients also termed the gadget-designers (*thaumasiourgoi*) mechanicians. Some of them practice their art on the principles of air pressure, like Hero in his *Pneumatika*, while others utilize sinews and cords to imitate the movements of living creatures, like Hero in his *Automata* and *Balances*; still others depend upon bodies floating on water, like Archimedes in his *On Floating Bodies*, or on water clocks, which seem to be connected with the theory of sundials, like Hero in his *On Water Clocks*. Finally, they also termed mechanicians the sphere-makers, who construct models of the heavens based on the even and circular motion of water.

Timaeus Cornford 62C, p. 262:

‘Heavy’ and ‘light’ may be most clearly explained by examining them together with the expressions ‘above’ and ‘below’.

Cornford, 63A–63E, pp. 263–264:

As to the source of these terms and the things to which they really apply and which have occasioned our habit of using the words to describe a division of the universe as a whole, we may arrive at an agreement, if we make the following supposition. Imagine a man in that region of the universe which is specially allotted to fire, taking his stand on the main mass towards which fire moves, and suppose it possible for him to detach portions of fire and weigh them in the scales of a balance. When he lifts the beam and forcibly drags the fire into the alien air, clearly he will get the smaller portion to yield to force more readily than the greater; for when two masses at once are raised aloft by the same power, the lesser must follow the constraint more readily than the greater, which will make more resistance; and so the large mass will be said to be ‘heavy’ and to tend ‘downwards’, the small to be ‘light’ and to tend ‘upwards’. Now this is just what we ought to detect ourselves doing here in our own region. Standing on the Earth, when we are trying to distinguish between earthy substances or sometimes pure earth, we are dragging the two things into the alien air by violence and against their nature; both cling to their own kind, but the smaller yields more readily to our constraint than the larger and follows it more quickly into the alien element. Accordingly we have come to call it ‘light’ and the

region into which we force it ‘above’ ; when the thing behaves in the opposite way, we speak of ‘heavy’ and ‘below’. Consequently, the relation of these things to one another must vary, because the main masses of the kinds occupy regions opposite to one another: what is ‘light’ or ‘heavy’ or ‘above’ or ‘below’ in one region will all be found to become, or be, contrary to what is ‘light’ or ‘heavy’ or ‘above’ or ‘below’ in the opposite region, or to be inclined at an angle, with every possible difference of direction. The one thing to be observed in all cases, however, is that it is the travelling of each kind towards its kindred that makes the moving thing ‘heavy’ and the region to which it moves ‘below’, while the contrary names are given to their opposites. So much for the explanation of these affections.

Early Physics and Astronomy: A Historical Introduction By Olaf Pedersen
Mechanica [?]

Pappus, *Mathematical Collection* VIII.1–5, Cohen and Drabkin, pp. 183–186:

The science of mechanics, my dear Hermodorus, is not merely useful for many important practical undertakings, but is justly esteemed by philosophers and is diligently pursued by all who are interested in mathematics, since it is fundamentally concerned with the doctrine of nature with special reference to the material composition of the elements in the cosmos. For it examines bodies at rest, their natural tendency, and their locomotion in general, not only assigning causes of natural motion, but devising means of impelling bodies to change their position, contrary to their natures, in a direction away from their natural places. In this the science of mechanics uses theorems suggested to it by a consideration of matter itself.

Now the mechanicians of Hero’s school tell us that the science of mechanics consists of a theoretical and a practical part. The theoretical part includes geometry, arithmetic, astronomy, and physics, while the practical part consists of metal-working, architecture, carpentry, painting, and the manual activities connected with these arts. One who has had instruction from boyhood in the aforesaid theoretical branches, and has attained skill in the practical arts mentioned, and possesses a quick intelligence, will be, they say, the ablest inventor of mechanical devices and the most competent master-builder. But since it is not generally possible for a person to master so many mathematical branches and at the same time to learn all the aforesaid

arts, they advise a person who is desirous of engaging in mechanical work to make use of those special arts which he has mastered for the particular ends for which they are useful.

The most important of the mechanical arts from the point of view of practical utility are the following. (1) The art of the *manganarii*, known also, among the ancients, as mechanics. With their machines they need only a small force to overcome the natural tendency of large weights and lift them to a height. (2) The art of the makers of engines of war, who are also called mechanics. They design catapults to fling missiles of stone and iron and the like a considerable distance. (3) The art of the contrivers of machines, properly so-called. For example, they build water-lifting machines by which water is more easily raised from a great depth. (4) The art of those who contrive marvelous devices. They too are called mechanics by the ancients. Sometimes they employ air pressure, as does Hero in his *Pneumatica*; sometimes ropes and cables to simulate the motions of living things, e.g., Hero in his works on *Automata* and *Balances*; and sometimes they use objects floating on water, e.g., Archimedes in his work *On Floating Bodies*, or water clocks, e.g., Hero in his treatise on that subject, which is evidently connected with the theory of the sun dial. (5) The art of the sphere makers, who are also considered mechanics. They construct a model of the heavens [and operate it] with the help of the uniform circular motion of water.

Now some say that Archimedes of Syracuse mastered the principles and the theory of all these branches. For he is the only man down to our time who brought a versatile genius and understanding to them all, as Geminus the mathematician tells us in his discussion of the relationship of the branches of mathematics. But Carpus of Antioch says somewhere that Archimedes of Syracuse wrote only one book on a mechanical subject, that on sphere-construction, but did not consider any of the other mechanical branches worthy of literary treatment. Now this wonderful man, a man so richly endowed that his name will be celebrated forever by all mankind, is extolled by most people for his achievement in mechanics. But his chief concern was the composition of works dealing with the principal matters of geometric and arithmetic theory, even those parts often held to be least important. Evidently he was so devoted to these branches that he did not permit himself to add to them anything extraneous. But

Carpus and others have made use of geometry as a basis for various arts, and properly so. For in aiding numerous arts geometry is in no wise harmed by the association with them. Since geometry is, so to speak, the mother of these arts, it is not harmed by aiding in the construction of engines or in the work of the master-builder, or by association with geodesy, horology, mechanics, and scene-painting. On the contrary, geometry obviously promotes these arts and is justly honored and glorified by them. Such, then, is the nature of mechanics, which is both a science and an art, and such are the parts into which it is divided. Now I consider it well to set forth more concisely, clearly, and rigorously than my predecessors have done, the most important theorems proved geometrically by the old writers on the subject of the motion of heavy bodies, as well as the theorems which I succeeded in discovering for myself. I cite as examples:

1. If a given weight is drawn by a given force on a horizontal plane, to find the force by which the weight will be drawn up a plane inclined to the horizontal at a given angle. This proposition is useful to those mechanicians who construct machines for lifting weights, for by adding a force of men to the force found to be theoretically required they may be confident that the weight will be drawn up;
2. Given two unequal straight lines to find two mean proportionals in continued proportion. By this theorem every solid figure may be augmented or decreased in any given ratio;
3. Given a wheel with a known number of cogs or teeth, to find the diameter of a second wheel to be engaged with the first and having a given number of teeth. This proposition is generally useful and in particular for machine makers in connection with the fitting of clogged wheels.

Each of these propositions will be elucidated in its proper place along with other propositions useful to the master-builder and the mechanician. But first let us discuss those things which have to do with the matter of centers of gravity.

We do not have to discuss at this time what is meant by the heavy and the light, what is the cause of the upward or downward tendency of bodies, and in fact what significance attaches to the terms up and down and by what limits each is bounded. These matters have been treated by Ptolemy in his *Mathematica*. But we should consider just

what we mean by the center of weight of a given body, for that is the fundamental element in the whole subject of centers of gravity on which depend all the other parts of mechanical theory. For the other theorems in this field can be clear, in my opinion, if this fundamental concept is clear Now we define the center of gravity of any given body as a point within the body such that, if we imagine the body to be suspended from that point, the body will be at rest, maintaining its original position without any tendency to turn. Statics

Jammer, Concepts of Force, 1962, p. 17:

The idea of force, in the prescientific stage, was formed most probably by the consciousness of our effort, spent in voluntary actions, as in the immediate experience of moving our limbs, or by the consciousness of the feeling of a resistance to be overcome in lifting a heavy object from the ground and carrying it from one place to another.