Numbers and fractions in Greek papyri

Jordan Bell

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1 Hieratic papyri

Rhind Mathematical Papyrus (RMP), BM 10057 and 10058. The translations I have consulted are Chace [10] and Clagett [11].

RMP $\frac{2}{n}$ entry for $\frac{2}{7}$ is the following [11, p. 122]:

> **1/4** [of 7 is] 1 1/2 1/4, **1/28** [of 7 is] 1/4.

	1	7			
	1/2	3 1/2		1	7
\	1/4	1 1/2 1/4		2	14
\	4	28	1/4	4	28.

$\frac{1}{4}$ of 7 is $1\,\overline{2}\,\overline{4}$. $1+\overline{2}+\overline{4}+R=2$. $R=\overline{4}$. $7\cdot 4=28$ so $\frac{1}{28}$ of 7 is $\overline{4}$.

$$2=\frac{1}{4}\cdot 7+R=\frac{1}{4}\cdot 7+\frac{1}{28}\cdot 7.$$

Then

$$\frac{2}{7}=\overline{4}\,\overline{28}.$$

Cf. Chace [10, pp. 14–15].

Chace [10, pp. 5–6]:

> Egyptian division might be described as a second kind of multiplication, where the multiplicand and product were given to find the multiplier. In the first kind of multiplication, the multiplier, being given, can be made up as a combination of the multipliers that were generally used, and the corresponding combination of products would be the required product. When it was the product that was given along with the multiplicand, various multipliers would be tried, 2, 10, and combinations of these numbers, or combinations of the fractions $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{10}$, and from the products thus obtained the Egyptians would endeavor to make up the entire given product. When they succeeded in doing this the corresponding combination of multipliers would be the required multiplier. But they were not

always able to get the given product at once in this way, and in such cases the complete solution of the problem involved three steps: (a) multiplications from which selected products would make a sum less than the required product but nearly equal to it; (b) determination of the remainder that must be added to this sum to make the complete product; and (c) determination of the multiplier or multipliers necessary to produce this remainder. The multipliers used in the first and third steps made up the required multiplier. The second step was called completion and will be explained below. For the third step they had a definite process which they generally used. The remainder, being a small number, would consist of one or more reciprocal numbers. For one of these numbers the third step may be expressed by the rule: To get the multiplier that will produce the reciprocal of a given whole number as a product multiply the multiplicand by the number itself and take the reciprocal of the result of this multiplication. If, for example, we wish to multiply 17 so as to get $\frac{1}{3}$, we take 3 times 17, which is 51, and then we can say that $\frac{1}{51}$ of 17 equals $\frac{1}{3}$.

	1		7
$\backslash$	$\overline{2}$	$\backslash$	$3\ \overline{2}$
	$\overline{\overline{3}}$		$4\ \overline{\overline{3}}$
$\backslash$	$\overline{3}$	$\backslash$	$2\ \overline{3}$
	$\overline{7}$		1
	$\overline{14}$		$\overline{2}$
$\backslash$	$\overline{42}$	$\backslash$	$\overline{6}$
	$\overline{2}\ \overline{3}\ \overline{42}$		$3\ \overline{2}\ 2\ \overline{3}\ \overline{6} = 3\ \overline{2}\ 2\ \overline{2} = 6$

Therefore $\frac{6}{7} = \overline{2}\ \overline{3}\ \overline{42}$.

Egyptian Mathematical Leather Roll (EMLR), BM 10250.

RMP Problem 69, 1120 : 80 = 14.

RMP 24: 19 : 8, RMP 25 16 : 3, RMP 21 4 : 15.

Gillings [16].

Parker, DMP Problem 56 [32].

2 Greek inscriptions

Rhodes and Osborne [34, p. xiii]:

> Temporary notices–lists of candidates for office, proposals for new legislation and so on–were written on whitewashed boards, and have not survived for us to read; for permanent publication bronze or wood was sometimes used, but the normal medium was stone. For example, texts of a city's religious calendars, of its laws and decrees, and of its alliances with other cities; schedules of work on a public

building project, and accounts of public expenditure on the project; inventories of precious objects in the temple treasuries or of ships in the dockyards; epigrams commemorating a famous victory; honours voted to a native or foreign benefactor; lists of office-holders and benefactors–all these and comparable documents might be inscribed on stone for members of the public to see.

Corpus des inscriptions de Delphes (CID) I.9 [34, p. 3], fifth/fourth century BC, Face A, ll. 19–23:

> Resolved by the Labyadai. On the tenth of the month Boukatios [Βουκατίου μηνὸς δεκ[ά]ται], in the archonship of Kampos, at the Assembly, by 182 votes [σὺμ πά[φ]οις ἑκατὸν ὀγδοήκοντ[α] δυοῖν].

CID I.9 [34, p. 5], Face B, ll. 30–34:

> Anyone who does not swear may not be a *tagos*. If someone serves as a *tagos* without swearing he is to pay a fine of 50 drachmas.
>
> If the seller is a slave-man or a slave-woman, he shall be beaten with fifty lashes with the whip [πληγὰς τῆι] by the *archontes* commissioned in each matter.

[34, p. 141], B(a), ll. 1–10:

> Of the cities these failed to pay the interest that they should have paid during our magistracy and did not pay during the four years: the people of Ceos 4,127 dr., $1\frac{1}{2}$ obols; the people of Myconos 420 dr.; the people of Syros 4,900 dr.; the people of Siphnos 2,089 dr. 2 obols; the people of Tenos 2,400 dr.; the Thermaians from Icaros 400 dr.; the people of Paros 4 talents 1,830 dr.; the Oinaians from Icaros 1 talent 80 dr. Of the cities these did not pay the interest during the four years of our magistracy during the archonships of Galleas, Gharisander, Hippodamas and Socratides at Athens and of Epigenes, Galaios, Hippias, and Pyrraethus on Delos: the people of Naxos, 1 talent 3,600 dr.; the people of Andros 2 talents; the people of Carystus 1 talent 2,400 dr.

Stephen Lambert, P. J. Rhodes: Payments from the treasury of Athena, 410/409 BC, *IG* I^3 375:

> In the seventh prytany, of Antiochis X, on the fifth of the prytany, was handed over to Dionysios of Kydathenaion and his fellow officials, for the two-obol grant, 1 tal; on the seventh of the prytany, to the hellenotamiai Thrason of Boutadai and his fellow officials, for the two-obol grant, 1 tal. 1,232 dr. $3\frac{1}{4}$ ob.; on the same day, to the hellenotamiai Phalanthos of Alopeke and his fellow officials, fodder for the horses, 4 tal. (?); on the sixteenth of the prytany, to the

> hellenotamiai Proxenos 25 of Aphidna and his fellow officials, 1,534 dr. 3 ob.; on the twenty-fourth of the prytany, to the hellenotamiai Eupolis of Aphidna and his fellow officials, 5,400 dr.; on the twenty-seventh of the prytany, to the hellenotamiai Kallias of Euonymon and his fellow officials, 1 tal. 2,565 dr. $4\frac{1}{2}$ ob.

Tod [42] and [43] on the acrophonic numerals.

3 Greek papyri

Hultsch [22, p. 170], symbols for numbers and fractions in Greek manuscripts.

Marrou [29]

Smyly [40, pp. 516–517]:

> The letters of the ordinary Greek alphabet, together with ϛ ϙ and ϡ, were arranged in four horizontal rows each of which contained nine symbols:

α	β	γ	δ	ε	ϛ	ζ	η	ϑ
ι	κ	λ	μ	ν	ξ	ο	π	ϙ
ρ	σ	τ	υ	φ	χ	ψ	ω	ϡ
͵Α	͵Β	͵Γ	͵Δ	͵Ε	͵Ϛ	͵Ζ	͵Η	͵Θ

> The symbols of the first row represent units (μονάδες) from 1 to 9; those of the second row tens (δεκάδες) from 10 to 90; those of the third hundreds (ἑκατοντάδες) from 100 to 900 and those of the fourth thousands (ξιλιάδες) from 1000 to 9000. The fourth row is a repitition of the first, the symbols being differentiated by a large curved flourish at the top, which makes them very prominent in papyrus documents; this prominence is best attained in modern printing by the employment of capital letters. Owing to the loss of all Greek treatises on elementary Logistic we are ignorant of the Greek names of these rows, but we learn from Martianus Capella VII, 745 that in Latin they were called *versus*: – primus igitur versus est a monade usque ad enneadem, secundus a decade usque ad nonaginta, tertius vero ab hecatontade usque ad nongentos, quartus qui et ultimus a mille usque ad novem milia, licet nonnulli Graeci etiam μύρια adiecisse videantur. The word is also found in Favonius Eulogius, *in somnium Scipionis*, p. 22: – primi versus absolutio novenario numero continetur.

Martianus Capella, *The Marriage of Philology and Mercury* [41, p. 286]:

> The first series [*versus*] runs from the monad to the ennead; the second from the decad to ninety; the third from one hundred to nine hundred; the fourth and last from one thousand to nine thousand; although some Greek writers appear to have included the myriad [μύρια].

Smyly [40, pp. 519–520]:

> When numbers were written in connexion with words they were often distinguished from the latter by a horizontal line drawn above them, $\overline{\pi\kappa\delta} = 124$. In the course of time this line was written with an upward inflection, so that it came to present the appearance of an accent in the form generally given in modern texts. This careless method of writing is inconvenient in the extreme, as it leads to great confusion between integers and fractions. In purely arithmetical operations the distinguishing line was unnecessary and accordingly was generally omitted.
>
> The space at my disposal does not permit me to discuss fully the treatment of fractions; but since they occur in some of the examples which I shall have to quote, I shall briefly indicate the various methods of writing them. Of these there were three: – 1) The fractions most often employed were those with unity for numerator; the denominator only was written down and they were distinguished from integers by accents: thus $\delta' = \frac{1}{4}$, $\iota'\beta' = \frac{1}{12}$ etc.; β' is an exception and denoted $\frac{2}{3}$, not $\frac{1}{2}$ for which a special sign $\angle$ was employed. These fractions were convenient in commercial transactions since the denominators chosen generally corresponded with the natural divisions of the measures or weights. 2) Occasionally vulgar fractions, such as are used now, were employed, the numerator being written on the line and the denominator either above it, or twice repeated after it; e.g. $\frac{12}{13}$ would have been written $\iota\beta^{\iota\gamma}$ or $\iota\beta\iota'\gamma'\iota'\gamma'$. 3) In astronomical calculations sexegesimal fractions were employed: they correspond to modern decimal fractions, but since the scale of notation was 60 instead of 10, they had the great advantage of being divisible by 3. From this system are derived the minutes and seconds which are still employed as fractions of an hour or a degree. The fraction $\frac{11}{12}$ might have been represented by 1) $\angle\ \gamma'\iota'\beta' = \frac{1}{2} + \frac{1}{3} + \frac{1}{12}$ or 2) $\iota\alpha\iota'\beta'\iota'\beta'$ or 3) $\nu\varepsilon' = \frac{55}{60}$.

Smyly [40, p. 522]:

> A symbol C was employed whenever it was necessary, for any reason, to separate one number from another; in subtraction it divided the subtrahend from the minuend; the remainder was introduced by ∩, a cursive corruption of Λ the initial letter of λοιπόν [adjective, “remaining over”]. Thus ͵Βφζ C ͵Αχπϑ ∩ ωιη is the Greek equivalent of $2507 - 1689 = 818$.

Smyly [40, p. 525]:

> Suppose that it is required to divide 889 by 24, the process would, in my opinion, have been written down thus: – ωπϑ C κδ ἐπὶ λ / ψκ ∩ ρξϑ C ἐπὶ ζ / ρξη ∩ α / λζ ἐλάσσω α.

> It is obvious that the first multiple of 24 must be 30; then $24 \times 30 = 720$, subtract from 889, the remainder is 169; the next figure of the quotient is seen to be 7, $24 \times 7 = 168$ $\therefore$ the result is 37 and there is a remainder 1.

Tod [44, p. 128]:

> In numbers from 1000 to 9999 the thousands, placed first in order, are represented by the same signs as the units, but their value is normally indicated by the addition to the letter of some mark of differentiation, most frequently a slanting stroke prefixed to the numeral; thus the number 1754 is written as /ΑΥΝΔ. But 10,000, 20,000 and higher multiples of 10,000 are never represented by /I, /K, etc.; instead the alphabetic system calls to its aid an acrophonic element, M, standing for μυριάς or μύριοι. In order to avoid confusion with M = 40, the single myriad may take the form of a monogram of MΥ, or it may have a small A placed immediately above it, denoting μία μυριάς. Multiples of a myriad are similarly represented by M with small letters superposed, showing the number of myriads represented; *e.g.* $\overset{\mathrm{B}}{\mathrm{M}} = 20{,}000$, $\overset{\mathrm{P}}{\mathrm{M}} = 1{,}000{,}000$.

Tod [44, p. 129]:

> The numeral signs used to represent any given number are normally arranged, as was invariably the case with acrophonic numerals, in the descending order of value, *e.g.* ρια′ = 111. This rule is, I believe, everywhere and always observed in numbers exceeding 1000, though in some parts of the Greek world examples of a reversed or mixed order in the representation of numbers below 1000 are very common.

Mithradates VI Eupator. No. 586 [46]: ΒΑΣΙΛΕΩΣ ΘΣ ΜΙΘΡΑΔΑΤΟΥ ΕΥΠΑΤΟΡΟΣ.

P. Brem. 36 [48, pp. 83–86] = Chrest. Wilck. 352 [47, pp. 414–415], [27, p. 114], 28 December AD 117:

> To Apollonios, strategos of the Seven-Village Apollinopolite nome, from Paphis son of Honēs and his brothers, of the village of Terythis. Near the said village district there are registered in the name of our father Honēs royal land as follows: taxed at [the rate of] $3\frac{1}{2}$ artabas [per aroura], $3\frac{1}{12}$ arouras [γ ι̅β̅ (ἄρουραι) γ∠], and another $1\frac{1}{2}$ arouras; at 2 artabas, $\frac{43}{64}$ aroura [ἄλ(λαι) δμοίως (ἄρουραι) α∠, καὶ ἀν(ὰ) β ∠η̅λ̅[̅β̅]̅ξ̅δ̅]; at $5\frac{1}{12}$ artabas, $\frac{1}{16}$ aroura [ἀν(ὰ) ε ι̅β̅ ι̅ϛ̅]; at $4\frac{1}{12}$ artabas, 1 aroura [ἀν(ὰ) δ ι̅β̅ (ἄρουρα) α]; total, $6\frac{47}{64}$ arouras [γί(νονται) (ἄρουραι) ϛ∠η̅ι̅ϛ̅λ̅β̅ξ̅δ̅].

P. Mon. Epiph. II.619 [14, p. 136], Monastery of Epiphanius, Thebes, uncertainly dated to sixth or seventh century AD,

πρωτη	ογδοη	πεμπτε
δευτερη	εννατη	και δεκα-
τριτη	δεκατη	τη
τεταρτη	ενδεκατη	
πεμπτη	δ[ω]δεκατη	
εκτη	[τρ]ις και δεκατη	
εβδομη	τε[σσα]ρες κ[α]ι	
	[δεκατ]η	

Smith, *A Dictionary of Greek and Roman Antiquities*, 1891, s.v. logistica.

Vogel [45]

Knorr [25]

Benoit, Chemla and Ritter [3]

Lahun Mathematical Papyri IV.2 [19], UC 32159, recovered in Kahun, dated about 1800 BC.

P. Sarga, Wadi Sarga, about 15 miles south of Asyut [13]

The Akhmim Mathematical Papyrus, P. Cairo Inv. 10758 [2]

Zalateo [50]

Chester Beatty codex Ac. 1390 [8, pp. 33–56], Dishna, ca. fourth century AD, Mathematical School Exercises in Greek. Extant Page Three, ll. 19–23 [8, pp. 54–55]:

> $3'$ $14'$ $42'$ (is) 200 talents [τὸ γ″ $\overline{\text{ιδμ}}$β (τάλαντα) Σ]. How much is the $42'$ part [πόσου τὸ $\overline{\text{μ}}$β μέρος]? Do it thus. In what ψῆφος does $3'$ $14'$ $42'$ [γ″ $\overline{\text{ιδμ}}$β] go? (It is) from (the table of) 7ths. 3/7 is $3'$ $14'$ $42'$ [ἀπὸ ζ τῶ(ν) γ τὸ $\overline{\text{ζ}}$ γ″ $\overline{\text{ιδμ}}$β]. Take the last part, $42'$ [$\overline{\text{μβ}}$]. $3'$ $14'$ $42'$ [γ″ $\overline{\text{ιδμ}}$β] (of 42 is) 18 [$\overline{\text{ιη}}$]. (Dividing) the aforementioned (200) talents by 18 [$\overline{\text{ιη}}$] equals 11 $9'$ talents [(τάλαντα) ια⟨ϑ′⟩]. Number (of talents) 466 2/3 [χξϛβ′].

Brashear [8, p. 53] remarks that there is not a satisfactory definition of the term ψῆφος (pebble, in particular a pebble used in reckoning), and that it has been translated by other editors variously as "calcul" (calculation, reckoning) and Tabelle (table).

$\overline{3}$ of 42 is 14. $\overline{14}$ of 42 is 3. Then $\overline{3}\,\overline{14}\,\overline{42}$ of 42 is $14 + 3 + 1 = 18$. On the one hand $\overline{3}\,\overline{14}\,\overline{42}$ of the sought quantity x is 200 talents, and on the other hand $\overline{3}\,\overline{14}\,\overline{42}$ of 42 is 18. Therefore the proportion 200 talents : 18 is the same as the proportion $x : 42$. 466 $\overline{\overline{3}}$, υξϛβ′, in Brashear's translation is right, 666 $\overline{\overline{3}}$, χξϛβ′, in the Greek text is wrong.

Crawford [12]

Louvre [5], [6], [9], [7]

T. Varie 7 = P. Vat. gr. 55 A [33, pp. 40–43], Oxyrhynchites?, seventh century AD. Table of seventeenths (Col. I) and table of nineteenths (Col. II). The table of seventeenths is the following:

τὸ ιζ [ἐν] ψήφων τνβ L $\overline{\text{γ ιζ λδ}}$ να τῆς μιᾶς τὸ ιζ ιζ
(οὕτως)·

τῶν β	ιβ να ξη	ιβ α γ′ $\overline{\text{ιβ}}$ να τὸ γ′ ξη τὸ d⁻
τῶν γ	ιβ ιζ να ξη	ιβ α γ′ $\overline{\text{ιβ}}$ ιζ α να τὸ γ′ ξη τὸ d
τῶν δ	ιβ ιε ιζ ξη πε′	ιβ α γ′ $\overline{\text{ιβ}}$ ιε α ι λ ιζ α ξη τὸ d πε τὸ ε
τῶν ε	d λδ ξη	d δ d λδ τὸ L ξη τὸ d⁻
τῶν ϛ	γ′ να	γ′ ε w να τὸ γ′
τῶν ζ	γ′ ιζ να	γ′ ε w ιζ α να τὸ γ′
τῶν η	γ′ ιε ιζ πε′	γ′ ε w ιε α ι λ ιζ α πε τὸ ε⁻
τῶν ϑ	L λδ′	L η L λδ τὸ L′
τῶν ι	L ιζ λδ′	L η L ιζ α λδ τὸ L
τῶν ια	L ιβ λδ′ να ξη⁻	L η L ιβ αγ′ $\overline{\text{ιβ}}$ λδ τὸ L να τὸ γ′ ξη τὸ d
τῶν ιβ	L ιβ ιζ λδ′ να ξη⁻	L η L ιβ αγ′ $\overline{\text{ιβ}}$ ιζ α λδ τὸ L να τὸ γ′ ξη τὸ d
τῶν ιγ	L d⁻ ξη⁻	L η L d⁻ δ d⁻ ξη τὸ d⁻
τῶν ιδ	L d⁻ ιζ ξη⁻	L η L d⁻ δ d⁻ ιζ α ξη τὸ d⁻
τῶν ιε	L γ′ $\overline{\text{λδ′ να}}$	L η L γ′ ε w λδ τὸ L να τὸ γ′
τῶν ιϛ	L γ′ $\overline{\text{ιζ λδ′}}$ να	L η L γ′ ε w ιζ α λδ τὸ L ν[α τὸ γ′]
τῶν ιζ	α	α ιζ

$\frac{6000}{17}$: $352\ \bar{2}\ \bar{3}\ \overline{17}\ \overline{34}\ \overline{51}$.
$\frac{1}{17}$: $\overline{17}$.
$\frac{2}{17}$: $\overline{12}\ \overline{51}\ \overline{68}$. $\overline{12} \cdot 17 = 1\ \bar{3}\ \overline{12}$. $\overline{51} \cdot 17 = \bar{3}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{3}{17}$: $\overline{12}\ \overline{17}\ \overline{51}\ \overline{68}$. $\overline{12} \cdot 17 = 1\ \bar{3}\ \overline{12}$. $\overline{17} \cdot 17 = 1$. $\overline{51} \cdot 17 = \bar{3}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{4}{17}$: $\overline{12}\ \overline{15}\ \overline{17}\ \overline{68}\ \overline{85}$. $\overline{12} \cdot 17 = 1\ \bar{3}\ \overline{12}$. $\overline{15} \cdot 17 = 1\ \overline{10}\ \overline{30}$. $\overline{17} \cdot 17 = 1$. $\overline{68} \cdot 17 = \bar{4}$. $\overline{85} \cdot 17 = \bar{5}$.
$\frac{5}{17}$: $\bar{4}\ \overline{34}\overline{68}$. $\bar{4} \cdot 17 = 4\ \bar{4}$. $\overline{34} \cdot 17 = \bar{2}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{6}{17}$: $\bar{3}\ \overline{51}$. $\bar{3} \cdot 17 = 5\ \bar{\bar{3}}$. $\overline{51} \cdot 17 = \bar{3}$.
$\frac{7}{17}$: $\bar{3}\ \overline{17}\ \overline{51}$. $\bar{3} \cdot 17 = 5\ \bar{\bar{3}}$. $\overline{17} \cdot 17 = 1$. $\overline{51} \cdot 17 = \bar{3}$.
$\frac{8}{17}$: $\bar{3}\ \overline{15}\ \overline{17}\ \overline{85}$. $\bar{3} \cdot 17 = 5\ \bar{\bar{3}}$. $\overline{15} \cdot 17 = 1\ \overline{10}\ \overline{30}$. $\overline{17} \cdot 17 = 1$. $\overline{85} \cdot 17 = \bar{5}$.
$\frac{9}{17}$: $\bar{2}\ \overline{34}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\overline{34} \cdot 17 = \bar{2}$.
$\frac{10}{17}$: $\bar{2}\ \overline{17}\ \overline{34}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\overline{34} \cdot 17 = \bar{2}$.
$\frac{11}{17}$: $\bar{2}\ \overline{12}\ \overline{34}\ \overline{51}\ \overline{68}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\overline{12} \cdot 17 = 1\ \bar{3}\ \overline{12}$. $\overline{34} \cdot 17 = \bar{2}$. $\overline{51} \cdot 17 = \bar{3}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{12}{17}$: $\bar{2}\ \overline{12}\ \overline{17}\ \overline{34}\ \overline{51}\ \overline{68}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\overline{12} \cdot 17 = 1\ \bar{3}\ \overline{12}$. $\overline{17} \cdot 17 = 1$. $\overline{34} \cdot 17 = \bar{2}$. $\overline{51} \cdot 17 = \bar{3}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{13}{17}$: $\bar{2}\ \bar{4}\ \overline{68}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\bar{4} \cdot 17 = 4\ \bar{4}$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{14}{17}$: $\bar{2}\ \bar{4}\ \overline{17}\ \overline{68}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\bar{4} \cdot 17 = 4\ \bar{4}$. $\overline{17} \cdot 17 = 1$. $\overline{68} \cdot 17 = \bar{4}$.
$\frac{15}{17}$: $\bar{2}\ \bar{3}\ \overline{34}\ \overline{51}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\bar{3} \cdot 17 = 5\ \bar{\bar{3}}$. $\overline{34} \cdot 17 = \bar{2}$. $\overline{51} \cdot 17 = \bar{3}$.
$\frac{16}{17}$: $\bar{2}\ \bar{3}\ \overline{17}\ \overline{34}\ \overline{51}$. $\bar{2} \cdot 17 = 8\ \bar{2}$. $\bar{3} \cdot 17 = 5\ \bar{\bar{3}}$. $\overline{17} \cdot 17 = 1$. $\overline{34} \cdot 17 = \bar{2}$. $\overline{51} \cdot 17 = \bar{3}$.
$\frac{17}{17}$: 1. $1 \cdot 17 = 17$.

P. Köln VII.325, Inv. 20839 C 2-21 v [20, pp. 166–174], Heracleopolites, end of sixth century/seventh century, nine arithmetic exercises. First example, ll. 1–4:

> ἀπὸ τῶν γw τὸ ξε γί(νεται) μόρ(αι) νβ ο πδ ϙα. ἄλλη μέϑωτος· ἀπὸ τ(ῶν) γw ἐπὶ γ γί(νεται) ια, ἀπὸ τ(ῶν) ξε ἐπὶ γ γί(νεται) ρϙε, ἀπὸ τ(ῶν)

ια ἐπὶ ζ γί(νεται) οζ, ἀπὸ τ(ῶν) ρϙε ἐπὶ ζ γί(νεται) ͵ατξε. τουτέστιν οζ εἰς ͵ατξε, γί(νεται) μόρ(ια) ν̅β̅ ⸗ κϛ d′ ο ⸗ ιϑ ∠ πδ ⟨ ⸗ ⟩ ιϛ d′ ϙ̅α̅ ⸗ [ιε]

$3\,\overline{\overline{2}}$: 65 makes $\overline{52}$ $\overline{70}$ $\overline{84}$ $\overline{91}$. Another method: $3\,\overline{\overline{2}} \cdot 3$ makes 11, $65 \cdot 3$ makes 195, $11 \cdot 7$ makes 77, $195 \cdot 7$ makes 1365. Result, 77 : 1365 makes $\overline{52}$ ⸗ 26 $\overline{4}$ $\overline{70}$ ⸗ 19 $\overline{2}$ $\overline{84}$ ⸗ 16 $\overline{4}$, $\overline{91}$ ⸗ 15.

w stands for a glyph used in the edition that looks like the Coptic letter shay, and denotes two thirds. ⸗ stands for a glyph used in the edition.

Second example, ll. 5–6:

τ(ῶν) ͵α τὸ ͵αα γί(νεται) μόρ(ια) ∠ γ ιγ κβ λ[γ] οζ [∠ φ∠]γ̅ τλγw ι̅γ̅ οζ
κ̅β̅ με∠ λ̅γ̅ λ γ′ ο̅ζ̅ ιγ

P. Oxy. IV.669 [17, pp. 116–121], Oxyrhynchus, end of the third century AD, Metrological Work, ll. 1–20:

The schoenium used in land-survey has 8 eighths [ὤγδοα η], and the eighth [ὄγδοον] has 12 cubits [πήχις ιβ], so that the schoenium used in land-survey has 96 cubits [πηχῶν ϙϛ], while the . . . schoenium has 100 cubits [πηχῶν ρ]. The linear cubit is that which is measured by length alone, the plane cubit is that which is measured by length and breadth; the solid cubit is that which is measured by length and breadth and depth or height. The . . . building cubit contains 100 plane cubits [[ἐμβαδικοὺς πή]χις ρ]. Ναύβια are measured by the ξύλον; the royal ξύλον contains 3 [γ] cubits, 18 [ιη] παλαισταί, 72 [οβ] δάκτυλοι, while the . . . ξύλον contains $2\frac{2}{3}$ cubits [ββ′], 16 [ιϛ] παλαισταί and 64 [ξδ] δάκτυλοι; so that the schoenium used in land-survey contains 32 [λβ] royal ξύλα and 36 [λϛ] . . . ξύλα.

ll. 31–41:

2 [β] παλαισταί make a λιχάς, 3 [γ] παλαισταί a σπιθαμή, 4 [δ] παλαισταί an (Egyptian?) foot [πούς], 5 [[ε]] a cloth-weaver's cubit [πῆχυς λινοῦφικὸς] . . . , 6 [ϛ] παλαισταί a public and a carpenter's cubit, 7 [[ζ]] παλαισταί a Nilometric cubit, 8 [η] παλαισταί a . . . cubit, 10 [ι] παλαισταί a βῆμα, which is the distance of the outstretched feet. 3 cubits [γ πήχ[εις]] make a public ξύλον, 4 [δ] cubits an ὀργυιά, which is the distance of the outstretched hands. . . cubits make a κάλαμος, $6\frac{2}{3}$ [ϛβ′] an ἄκαινα.

Hunt and Edgar [23, pp. 2–3], No. 1, P. Eleph. 1, Marriage Contract, 311 BC, ll. 1–2:

Ἀλεξάνδρου τοῦ Ἀλεξάνδρου βασιλεύοντος ἔτει ἑβδόμωι, Πτολεμαίου σατραπεύοντος ἔτει τεσαρεσκαιδεκάτωι, μηνὸς Δίου.

In the 7th year of the reign of Alexander son of Alexander, the 14th year of the satrapship of Ptolemy, in the month Dius.

Hunt and Edgar [23, pp. 4–7], No. 2, P. Tebt. 104, Marriage Contract. Hunt and Edgar divide this into a summary, ll. 1–4, and text of the contract, written in a second hand. Summary:

> Year 22, Mecheir 11 [(Ἔτους) κβ Μεχ(εὶρ) ια]. Philiscus son of Apollonius, Persian of the Epigone, acknowledges to Apollonia also called Kellauthis, daughter of Heraclides, Persian, having with her as guardian her brother Apollonius, that he has received from her in copper money 2 talents 4000 drachmae [(τάλαντα) β καὶ (δραχμὰς) ͵Δ], the dowry for herself, Apollonia, agreed upon with him . . . Keeper of the contract: Dionysius.

In the text of the contract, ll. 5–13:

> In the 22nd year [ἔτους δευτέρου καὶ εἰκοστοῦ] of the reign of Ptolemy also called Alexander, the god Philometor, the priest of Alexander and the other priests being as written in Alexandria, the 11th of the month Xandicus, which is the 11th of Mecheir [μηνὸς Ξανδικ[ο]ῦ ἑνδεκάτηι Μ[εχεὶ]ρ ἑνδεκάτηι], at Kerkeosiris in the division of Polemon of the Arsinoite nome. Philiscus son of Apollonius, Persian of the Epigone, acknowledges to Apollonia, also called Kellauthis, daughter of Heraclides, Persian, having with her as guardian her brother Apollonius, that he has received from her in copper money 2 talents 4000 drachmae [τάλαντα δύο καὶ δραχμὰς τετρακισχιλία[ς]], the dowry for herself, Apollonia, agreed upon with him.

Hunt and Edgar [23, pp. 10–11], No. 3, B.G.U. 1052, 13 BC, Marriage Contract, l. 34:

> (ἔτους) ιζ Καίσαρος Φαρμοῦθι $\overline{\kappa}$.
>
> The 17th year of Caesar, Pharmouthi 20.

Hunt and Edgar [23, pp. 96–99], No. 32, P. Oxy 95, AD 129, Sale of a Slave, ll. 1–3:

> The 13th year [Ἔτους τρισκαιδεκάτου] of the Emperor Caesar Trajanus Hadrianus Augustus, Pauni 29 [Παῦνι $\overline{\kappa\vartheta}$], at Oxyrhynchus in the Thebaid.

Hunt and Edgar [23, pp. 156–157], No. 52, P. Ryl. 157, AD 135, Division of Property Held on Lease, ll. 3–11:

> We acknowledge that we have divided between each other at this present time the domain-land vineyard which we hold on lease in the village Thrage in the toparchy of the Upper Suburb, being part of the holding of Xenon . . . whatever the extent of its acreage is, containing an orchard, and that Soeris also called Souerous has been

> allotted the southern portion, having forthwith paid to Eudaemonis for the choice 210 silver drachmae [ἀργυρίου δραχμὰς διακοσίας δέκα]. Its measurements are ... beginning from south to north inside the wall of the plot in the second stade following the western wall 1 schoenion, from the western wall eastward for a certain distance $1\frac{55}{64}$ schoenia [σχοινίον ἓν ἥ[μισ]υ τέταρτον ἑκκαιδέκατον δυοτριακοστὸν [τετρακαιεξηκ]οστόν], and from this latter boundary turning off to the north $\frac{23}{32}$ of a schoenion [σχοινίου ἥμισυ ὄγδο[ο]ν [ἑ]κκαιδέκατον δυοτρια[κοστὸν]], and from this boundary eastward up to the eastern wall which is the boundary of the whole plot $\frac{13}{23}$ of a schoenion [σχ[ο]ινίου τέταρτον ὄγδοον δυοτριακοστόν].

Hunt and Edgar [23, pp. 172–173], No. 57, P. Oxy 270, AD 94, Indemnification of a Surety, ll. 17–27:

> in the area of Seruphis from the holding of Demetrius the Milesian $3\frac{1}{2}$ arurae [ἀρούραις τρισὶ ἡμίσει] of catoecic and purchased land, and from the same holding out of 12 arurae [ἀρουρῶν δέκα δύο] of catoecic and purchased land the 5 arurae [ἀρουραῖς πέντε] remaining after the 7 [ἀρούρας ἑπτὰ] which she mortgaged to Taaphunehis daughter of Thonion, and from the holding of Callias a third share [τρίτῳ μέρει] of 8 arurae [ἀρουρῶν ὀκτό] of catoecic and purchased land, making $2\frac{2}{3}$ arurae [ἄρουραι δύο δίμοιρον], and in the area of Syron Kome from the holding of Heracleides together with that of Alexander $6\frac{3}{4}$ arurae [ἀρούραις ἓξ ἡμίσει τετάρτῳ] of catoecic land, and from the holding of Alexander and others $6\frac{1}{2}$ arurae [ἀρούραις ἓξ ἡμίσει] of catoecic and purchased land, making a total of $24\frac{5}{12}$ arurae [ἀρούραις εἴκοσι τέσσαροι τρίτῳ δωδεκατῳ] of catoecic land and land purchased for conversion into catoecic

Hunt and Edgar [23, pp. 186–189], No. 63, P. Ryl. 177, AD 246, Loan on Mortgage, ll. 4–5:

> We acknowledge that we have received from you by hand out of your house a loan at interest ... of one thousand nine hundred and twenty silver drachmae, total 1920 silver dr. [ἀργυρίου δραχμὰς χειλίας ἐνναкοσίας εἴκοσι, γ(ίνονται) ἀργ(υρίου) (δραχμαὶ) Ἀϡκ]

Hunt and Edgar [23, pp. 270–273], No. 89, P.S.I. 333, 256 BC, From Promethion to Zenon:

> Promethion to Zenon greeting. I suffered anxiety when I heard of your long protracted illness, but now I am delighted to hear that you are convalescent and already on the point of recovery. I myself am well. I previously gave your agent Heraclides 150 drachmae in silver [ἀργυρίου (δραχμὰς) $\overline{\rho\nu}$] from your account, as you wrote to me to do, and he is bringing you now 10 *hins* [ἵνια ῑ] of perfume in

> 21 vases [[ἀλ]αβάστροις $\overline{\kappa\alpha}$] which have been sealed with my finger-ring. For though Apollonius wrote to me to buy and give him also 300 [τ] wild pomegranate wreaths, I did not manage to give him these at the same time, as they were not ready, but Pa . . . will bring them to him at Naucratis; for they will be finished before the 30th [λ]. I have paid the price both of these and of the perfume from your account, as Apollonius wrote. I have also paid a charge of 10 drachmae [(δραχμὰς ι] in copper for the boat in which he is sailing up. And 400 drachmae in silver [ἀργυρίου (δραχμαὶ) υ] have been paid to Iatrocles for the papyrus rolls which are being manufactured in Tanis for Apollonius. Take note then that these affairs have been settled thus. And please write yourself if ever you need anything here. Goodbye. Year 29, Choiach 28 [(ἐ΄τους) κϑ Χοίαχ $\overline{\kappa\eta}$]. (Addressed) To Zenon. (Docketed) Year 29, Peritius 3 [(ἐ΄τους) κϑ Περιτίου γ]. Promethion about what he has paid.

Hunt and Edgar [24, pp. 406–409], No. 346, P.S.I. 488, 257 BC, Tender for Repairing Embankments:

> To Apollonius the dioecetes greeting from Harmais. At the city of Memphis the various embankments measure 100 schoenia [σχοινίων ρ], being as follows: those of the Syro-Persian quarter 12 schoenia [σχοινίων $\overline{\iota\beta}$], of Paasu 7 [$\overline{\zeta}$], those above the quay of Hephaestus and those below 4 [$\overline{\delta}$], those about the city together with the palace 23 [$\overline{\kappa\gamma}$], those of the Carian quarter . . . , of the Hellenion 3 [$\overline{\gamma}$], beyond Memphis those on the west of the royal garden 20 [$\overline{\kappa}$] and on the east . . . and on the north 5 schoenia 30 cubits [$\overline{\varepsilon}$ (πηχῶν) $\overline{\lambda}$]. For the heaping up of these embankments the sum given in the 28th year [$\overline{\kappa\eta}$ (ἔτει)] was 1 talent 5500 drachmae [(τάλαντον) α (δραχμαὶ) ͵Εφ], when the rise of the river was 10 cubits 3 palms $1\frac{1}{6}$ finger-breadths [πη(χῶν) ι πα(λαιστῶν) γ δα(κτύλου) αϛ΄], and in the 27th year [κζ (ἔτει)] the sum given was 1 talent 1300 drachmae [(τάλαντον) α (δραχμαὶ) ͵Ατ], when the river rose 10 cubits 6 palms $2\frac{2}{3}$ finger-breadths [πή(χεις) ι πα(λαιστὰς) ϛδα(κτύλους) ββ΄]. I now undertake to heap up the same embankments beginning from their bases to the height of a rise of 12 cubits [πη(χῶν) ιβ], to the satisfaction of the oeconomus and the chief engineer, if I receive 1 talent [(τάλαντον) α] from the Treasury. And according to the usual practice we shall be furnished with mattocks, which we will return. Farewell.

Hunt and Edgar [24, pp. 424–425], No. 354, P. Giess. 4, AD 118, Offer to Lease State Lands at a Reduced Rate:

> As our lord Hadrianus Caesar among his other indulgences has ordained that Crown land, public land, and domain land shall be cultivated at rents corresponding to their various values and not in accordance with the old order, and as we have been overburdened

for a long time with public dues on Crown land in the area of the metropolis, Pseathuris the younger paying on $8\frac{1}{2}$ [η∠] arurae at the rate of $2\frac{1}{12}$ artabae [β ι′β′] for each and on $\frac{7}{32}$ [η′ ι′ϛ′ λ′β′] of an arura at the rate of $3\frac{1}{12}$ [γ ι′β′], and Senpachompsais daughter of Pseathuris on $1\frac{11}{16}$ [α∠ η′ ι′ϛ′] arurae at the rate of $4\frac{1}{12}$ [δ ι′β′] artabae, total $10\frac{3}{8}$ [ι δ′ η′] arurae, having just now obtained the indulgence mentioned we present this application, undertaking to cultivate the aforesaid $10\frac{3}{8}$ [ι δ′ η′] arurae at the rate of $1\frac{1}{24}$ [α κ′δ′] artabae of wheat for each arura, unirrigated land and half of the artificially irrigated land being exempted according to custom.

P. Hibeh 87 [18, p. 250], Hibeh, 256/5 BC, Advance of Seed-Corn:

> . . . son of Heraclides and Her . . . son of Meniscus and Ze . . . son of . . . , holders of 25 arourae [(εἰκοσιπεντάρουροι) >'eqein], acknowledge that we have received from . . . , sitologus, for the holdings which we possess at the village of the Pastophori, as seed for the 30th year [λ (ἔτος)] $79\frac{3}{4}$ artabae of wheat and $33\frac{1}{4}$ artabae of barley [πυρ[ο]ῦ ἑβδομήκ[ον]τα ἐννέα ἥμυσυ τέταρτον καὶ κριθῆς τριάκοντα τρεῖς τέταρτον], in pure corn measured by the receiving measures, and we make no complaint.

Hibeh [15]

P. Yale I.75, Inv. 297 [30, p. 239], Tebtunis, AD 176, Two Customs House Receipts From Tebtunis:

> Paid through the gate of Tebtunis, the 1/100 and 1/50 [π′ καὶ ν′], by Petesouchos, importing a donkey, female, black, having shed its teeth, one. Year 16 [(ἔτους) ιϛ], Payni twenty-one, 21 [Παῦνι μία καὶ εἰκάδι κα]. Seal: Year 16 [(ἔτους) ιϛ] of Aurelius Antoninus Caesar the lord, the gate of Tebtunis.

Galen, *De antidotis*, book I, chapter V [1, p. 142], [26, pp. 31–32]:

> Books lying in the libraries that have signs for numbers are easily distorted, with the five changing into nine, and also the seventy, or the 13, through the addition of one letter, just as also through the subtraction of another. As a result I follow the practice of Menecrates author of the work entitled Αὐτοκράτωρ ὁλογράμματος, in which the 7's were written out with two syllables, not ζ by itself; the 20's with three syllables, not κ by itself; and the 30's with four syllables, not λ by itself – and the rest similarly, as I myself shall do as well. I also praise Andromachus who wrote his *Theriaka* in verse, as did some others. Damocrates, too, did rightly by writing recipes in verse, for then the rascals are least of all able to distort them.

P. Princ. Inv. GD 9556 [39, p. 246] = SB XX.15071 [36, pp. 637–638], provenance unknown, third or fourth century AD.

P. Petrie, Flinders Petri papyri [28]
P. Petrie 3.76 [40, p. 521]:

ʼϠψξηβʹ ʼΒρξϛ∠γʹ ʼΔσϟδηʹ ʼΖψκϛηʹ ʼΑτξϑβʹ ʼϠχιεδʹ ʼΗωνηδʹ ʼΑωπ∠
$\overset{\alpha}{\mathrm{M}}$ʼΓσκγβʹ ʼΑρπδ∠γʹ ʼΔϟϛ∠γʹ ʼΑχοεδʹ / $\overset{\varepsilon}{\mathrm{M}}$ ʼΘωκ

	βʹ+∠+γʹ+ηʹ+ηʹ+βʹ+δʹ+δʹ+∠+βʹ+∠+γʹ+∠+γʹ+δʹ
=	ϛ
	η+ϛ+δ+ϛ+ϑ+ε+η+γ+δ+ϛ+ε+ϛ
=	ο
	ξ+κ+ϟ+κ+ξ+ι+ν+π+κ+π+ϟ+ο+ο
=	ψκ
	ψ+ρ+σ+ψ+τ+χ+ω+ω+σ+ρ+χ+ψ
=	ʼΕω
	ʼϠ+ʼΒ+ʼΔ+ʼΖ+ʼΑ+ʼϠ+ʼΗ+ʼΑ +ʼΓ+ʼΑ+ʼΔ+ʼΑ+ʼΕ
=	$\overset{\delta}{\mathrm{M}}$ʼΘ
	$\overset{\alpha}{\mathrm{M}}$+$\overset{\delta}{\mathrm{M}}$
=	$\overset{\varepsilon}{\mathrm{M}}$
	$\overset{\varepsilon}{\mathrm{M}}$ʼΘωκ

Leonardo Pisano, *Liber abaci*, chapter 5 [37, p. 49], [4, p. 24]:

> If over any number will be made a fraction line, and over the same line will be written another number, the upper number means the number of parts determined by the lower number; the lower is called the denominator and the upper is called the numerator. And if over the number two will be made a fraction line, and over the fraction line the number one is written, then one of the two parts of the whole is meant, that is, one half, thus $\frac{1}{2}$, and if over the number three the same one is put, thus $\frac{1}{3}$, it denotes one third; and if over seven, thus $\frac{1}{7}$, one seventh; and if over 10, one tenth; and if over 19, a nineteenth part of the whole is meant, and so on successively. Also if two over three will be shown, thus $\frac{2}{3}$, two of three parts of the whole is meant, that is two thirds. And if over 7, then two sevenths, thus $\frac{2}{7}$, and if over 23, then two twenty-thirds will be denoted, and so on successively. Also if seven is put over nine, thus $\frac{7}{9}$, seven ninths of the whole is meant; and if 7 is put over 97, seven ninety-sevenths will be denoted. Also 13 put over 29 means thirteen twenty-ninths. And if 13 is put over 347, thirteen three hundred forty-sevenths will be indicated, and thus it is understood for the remaining numbers.
>
> Cum super quemlibet numerum quedam virgula protracta fuerit, et super ipsam quilibet alius numerus descriptus fuerit, superior numerus partem vel partes inferioris numeri affirmat; nam inferior denominatus, et superior denominans appellatur. Ut si super binarium

protracta fuerit virgula, et super ipsam unitas descripta sit, ipsa unitas unam partem de duabus partibus unius integri affirmat, hoc est medietatem sic $\frac{1}{2}$ et super ternarium ipsa unitas posita fuerit sic $\frac{1}{3}$, denotat tertium: et si super septenarium sic $\frac{1}{7}$ septimam; et si super 10 decimam; et si super 19, nonamdecimam partem unius integri affirmat, et sic deinceps. Item si binarius super ternarium extiterit sic $\frac{2}{3}$, duas partes de tribus partibus unius integri affirmat, hoc est duas tertias. Et si super 7 super septimas sic $\frac{2}{7}$ et si super 23 duas vigesimas tertias denotabunt, et sic deinceps. Item si septenarius super novenarium positus fuerit sic $\frac{7}{9}$ septem, novenas unius integri affirmant; et si 7 super 97, septem nonagesimas septimas denotabunt. Item 13 posita super 29, tredecim vigesimas nonas affirmant. Et si 13 sunt super 347, tredecim trecentesimas quadragesimas septimas indicabunt, et sic de reliquis numeris est intelligendum.

Leonardo Pisano, *Liber abaci*, chapter 5 [37, p. 50]:

If under a certain fraction line one puts 2 and 7, and over the 2 is 1, and over the 7 is 4, as here is displayed, $\frac{1\ \ 4}{2\ \ 7}$, four sevenths plus one half of one seventh are denoted. However if over the 7 is the zephir [*zephyrum*], thus $\frac{1\ \ 0}{2\ \ 7}$, one half of one seventh is denoted. Also under another fraction line are 2, 6, and 10; and over the 2 is 1, and over the 6 is 5, and over the 10 is 7, as is here displayed, $\frac{1\ \ 5\ \ 7}{2\ \ 6\ \ 10}$, the seven that is over the 10 at the head of the fraction line represents seven tenths, and the 5 that is over the 6 denotes five sixths of one tenth, and the 1 which is over the 2 denotes one half of one sixth of one tenth, and thus singly, one at a time, they are understood; ...

Leonardo Pisano, *Liber abaci*, chapter 7, part 6, first distinction [37, p. 119]:

In the first and second part of this chapter we taught how to add together several fractions into a single fraction. In this part truly we teach how to separate fractions with several parts into the sum of unit fractions, and seeing the parts of any fraction, to know the values of the part or parts of the integer one. This work is indeed divided into seven distinctions, the first of which is when the greater number which is below the fraction is divisible by the lesser, namely by that which is over the fraction line. The rule for the first distinction is that you divide the greater by the lesser, and you will have the part that the lesser is of the greater. For example, we wish to know what part $\frac{3}{12}$ is of the integer one. The 12 is indeed divided by the 3; this yields 4 for which you say $\frac{1}{4}$, and such is the part $\frac{3}{12}$ is of the integer one.

$\frac{k}{kl} = \frac{1}{l}$
Leonardo Pisano, *Liber abaci*, chapter 7, part 6, second distinction [37, p. 119]:

> The second distinction is when the greater number is not divisible by the lesser, but of the lesser can be made such parts which will divide integrally into the greater; in the rule for this distinction you make parts of the lesser by which you can divide the greater; and the greater is divided by each of the parts, and you will have unit fractions that the lesser makes from the greater.

$\frac{k+l}{klm} = \frac{1}{lm} + \frac{1}{km}$
Leonardo Pisano, *Liber abaci*, chapter 7, part 6, third distinction [37, pp. 121–122]:

> The third distinction indeed is when one more than the greater number is divisible by the lesser; the rule for this distinction is, you divide the number that is one more by the lesser, and the quotient of the division will be the part of the integer one, and will be less than the greater, and to this you add the same part of the part that is the greater number. For example, we wish to make unit fractions of $\frac{2}{11}$; that is from this distinction because one plus the 11, namely 12, is divisible by the 2 that is over the fraction; from this division comes the quotient 6 which yields $\frac{1}{6}$, and to this is added a sixth of an eleventh, namely $\frac{1\ 0}{6\ 11}$, for the unit fraction parts of $\frac{2}{11}$; using the same rule for $\frac{3}{11}$ you will have a quarter and $\frac{1\ 0}{4\ 11}$,that is $\frac{1}{44}\ \frac{1}{4}$. And for $\frac{4}{11}$ you will have a third and $\frac{1\ 0}{3\ 11}$, that is $\frac{1}{33}\ \frac{1}{3}$; and so for the $\frac{6}{11}$ you will have half and $\frac{1\ 0}{2\ 11}$, that is $\frac{1}{22}\ \frac{1}{2}$; and similarly for the $\frac{5}{19}$, as the 5 that is over the 19 is $\frac{1}{4}$ of 20, that is 1 plus the 19, you will have $\frac{1\ 0}{4\ 19}$, that is $\frac{1}{76}\ \frac{1}{4}$; still by the third distinction there are those that are composed a second time, as $\frac{2\ 0}{3\ 7}$, that is $\frac{1\ 0}{2\ 7}$ and $\frac{1\ 0}{6\ 7}$; as $\frac{2}{3}$ of $\frac{1}{6}\ \frac{1}{2}$; similarly $\frac{4\ 0}{7\ 9}$ is $\frac{1\ 0}{2\ 9}$ and $\frac{1\ 0}{14\ 9}$, because $\frac{4}{7}$ is $\frac{1}{14}\ \frac{1}{2}$; therefore the $\frac{3\ 0}{11\ 7}$ is $\frac{1\ 0}{4\ 7}$ plus $\frac{1\ 0}{44\ 7}$; similarly, the $\frac{3\ 0}{7\ 8}$ is reversed to $\frac{3\ 0}{8\ 7}$, that is from two composed distinctions, namely from the second and from the third.

$\frac{k}{kl-1} = \frac{1}{l} + \frac{1}{kl-1}$
Leonardo Pisano, *Liber abaci*, chapter 7, part 6, fourth distinction [37, pp. 122–123]:

> The fourth distinction is when the greater is a prime number, and the greater plus one is divisible by the lesser minus 1, as $\frac{5}{11}$ and $\frac{7}{11}$; this distinction rule is, you subtract 1 from the lesser, from which you make a unit fraction, namely with whatever is the number which is under the fraction, and then there will remain for you the parts using the third distinction; if you will subtract $\frac{1}{11}$ from $\frac{5}{11}$, then there will remain $\frac{4}{11}$, for which $\frac{4}{11}$ you will have the unit fractions $\frac{1}{33}\ \frac{1}{3}$ by the third distinction, and with the abovewritten $\frac{1}{11}$ added this will yield $\frac{1}{33}\ \frac{1}{11}\ \frac{1}{3}$; and by the same rule for $\frac{7}{11}$ you will have $\frac{1}{22}$ $\bar{1}11$ $\bar{1}2$, and for $\frac{3}{7}$ you will have $\frac{1}{28}\ \frac{1}{7}\ \frac{1}{4}$, for $\frac{6}{19}$ you will have $\frac{1}{76}\ \frac{1}{19}\ \frac{1}{4}$, and for $\frac{7}{29}$ you will have $\frac{1}{5}\ \frac{1}{29}\ \frac{1}{145}$; $\frac{1}{145}\ \frac{1}{29}\ \frac{1}{5}$ that is.

Pack [31], nos. 2306–2325

MPER XV.144 = P. Vindob. G 26011o [21, p. 134], Arsinoites/Heracleopolites, seventh century AD:

α β γ δ ε ϛ ζ η ϑ ι ια ιβ	1 2 3 4 5 6 7 8 9 10 11 12
ιγ ιδ ιε ιϛ ιζ ιη ιϑ ϰ ϰα ϰβ ϰγ	13 14 15 16 17 18 19 20 21 22 23
ϰδ ϰε ϰϛ ϰζ [ϰ]η [ϰ]ϑ [λ]	24 25 26 27 28 29 30
λ[α] λβ λγ [λδ λε λ]ϛ λζ [λη]	31 32 33 34 35 36 37 38
λϑ μ μα [μ]β [μ]γ μδ [με μϛ]	39 40 41 42 43 44 45 46
μζ μη μϑ ν [να ν]β [ν]γ νδ νε [νϛ]	47 48 49 50 51 52 53 54 55 56
νζ νη νϑ ξ ξα ξ[β ξ]γ ξδ ξε	57 58 59 60 61 62 63 64 65
ξϛ ξζ ξη ξϑ ο οα οβ	66 67 68 69 70 71 72
ογ [ο]δ οε οϛ οζ οη οϑ π π[α]	73 74 75 76 77 78 79 80 81
πβ πγ πδ πε πϛ π[ζ]	82 83 84 85 86 87
πη πϑ ϙ ϙ[α] ϙβ ϙγ ϙδ ϙε	88 89 90 91 92 93 94 95
ϙϛ ϙζ ϙη ϙϑ ρ ς τ υ φ χ	96 97 98 99 100 200 300 400 500 600
ψ ω ϡ ͵α ͵β ͵γ ͵δ ͵ε ͵ϛ ͵ζ ͵η ͵ϑ $\overset{\alpha}{\mu}$	700 800 900 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

P. Mich. XV.686, Inv. 5663a [38, pp. 2–7], Karanis, second or third century AD. Fragment B, Col. I:

τὸ μ′ τ[ῶν] ν αd′	the $\frac{1}{40}$ of the 50, 1 $\frac{1}{4}$
τὸ ν τῶν ξ αε′	the $\frac{1}{50}$ of the 60, 1 $\frac{1}{5}$
τὸ ξ′ τῶν ο αϛ′	the $\frac{1}{60}$ of the 70, 1 $\frac{1}{6}$
τὸ ο′ τῶν π αζ′	the $\frac{1}{70}$ of the 80, 1 $\frac{1}{7}$
τὸ π′ τῶν ϙ αη′	the $\frac{1}{80}$ of the 90, 1 $\frac{1}{8}$
τὸ ϙ τῶν ρ αϑ′	the $\frac{1}{90}$ of the 100, 1 $\frac{1}{9}$
τὸ ρ′ τῶν ρι αι′	the $\frac{1}{100}$ of the 110, 1 $\frac{1}{10}$

Fragment B, Col. II, multiples of $\overline{30}$:

τὰ γ ι′	3, $\overline{10}$
τὰ γ∠ ι′ξ′	3 $\overline{2}$, $\overline{10}$ $\overline{60}$
τὰ δ ι′λ′	4, $\overline{10}$ $\overline{30}$
[τὰ] δ∠ ι′ϰ′	4 $\overline{2}$, $\overline{10}$ $\overline{20}$

Fragment C, Col. I, multiples of $\overline{30}$:

[τ]ὰ ε ϛ′	5, $\overline{6}$
τὰ ε∠ ϛ′ξ′	5 $\overline{2}$, $\overline{6}$ $\overline{60}$
τὰ ϛ ε′	6, $\overline{5}$
τὰ ϛ∠ ε′ξ′	6 $\overline{2}$, $\overline{5}$ $\overline{60}$
[τὰ] ζ ε′λ′	7, $\overline{5}$ $\overline{30}$
[τὰ] ζ∠ ∠ε′[λ′ξ′]	7 $\overline{2}$, $\overline{2}$ [!] $\overline{5}$ $\overline{30}$ $\overline{60}$
τὰ η [ε′ ιε′]	8, $\overline{5}$ $\overline{15}$
τὰ η∠ ε′[ι′ξ′]	8 $\overline{2}$, $\overline{5}$ $\overline{10}$ [!] $\overline{60}$
τὰ ϑ ε′[ι′]	of 9, $\overline{5}$ $\overline{10}$
τὰ ϑ∠ ε′[ι′ξ′]	of 9 $\overline{2}$, $\overline{5}$ $\overline{10}$ $\overline{60}$
τὰ ι γ′	of 10, $\overline{3}$

Fragment C, Col. II, multiples of $\overline{30}$:

τὰ ιϑ [∠ι′λ′]	19, $\overline{2}$ $\overline{10}$ $\overline{30}$
τὰ ιϑ∠ ∠ι′κ′	19 $\overline{2}$, $\overline{2}$ $\overline{10}$ $\overline{20}$
τὰ κ β′	20, $\overline{\overline{3}}$
τὰ κ∠ β′ξ′	20 $\overline{2}$, $\overline{\overline{3}}$ $\overline{60}$
τὰ κα β′λ′	21, $\overline{\overline{3}}$ $\overline{30}$
τὰ κα∠ β′κ′	21 $\overline{2}$, $\overline{\overline{3}}$ $\overline{20}$
[τὰ] κβ β′ιε′	22, $\overline{\overline{3}}$ $\overline{15}$
[τὰ] κβ∠ β′ιβ′	22 $\overline{2}$, $\overline{\overline{3}}$ $\overline{12}$
[τὰ] κγ β′ι′	23, $\overline{\overline{3}}$ $\overline{10}$
[τ]ὰ κγ∠ β′ι′ξ′	23 $\overline{2}$, $\overline{\overline{3}}$ $\overline{10}$ $\overline{60}$
[τ]ὰ κδ β′ι′λ′	24, $\overline{\overline{3}}$ $\overline{10}$ $\overline{30}$
τὰ κδ∠ [β′λ′]	24 $\overline{2}$, $\overline{\overline{3}}$ $\overline{30}$ [!]
τὰ κε [β′ϛ′]	25, $\overline{\overline{3}}$ $\overline{6}$
τὰ κε∠ [β′ϛ′ξ′]	25 $\overline{2}$, $\overline{\overline{3}}$ $\overline{6}$ $\overline{60}$
τὰ κϛ β′[ε′]	26, $\overline{\overline{3}}$ $\overline{5}$
τὰ κϛ∠ β′[ε′ξ′]	26 $\overline{2}$, $\overline{\overline{3}}$ $\overline{5}$ $\overline{60}$

P. Mich. III.145, Inv. 4966, second century AD, unknown origin [49, pp. 34–52]. Robins describes the papyrus as composed of 19 pieces grouped into ten fragments. Fragment I has five columns but only Col. i–ii are intact. P. Mich. III.145, I,i [49, p. 36]:

της α κ′γ′	of 1, $\overline{23}$
[των β] ι′β′ σο′ϛ′	of 2, $\overline{12}$ $\overline{276}$
[των γ] ι′ μ′ϛ′ ρ′ι′ε′	of 3, $\overline{10}$ $\overline{46}$ $\overline{115}$
[των δ ϛ]′ ρ′λ′η′	of 4, $\overline{6}$ $\overline{138}$
[των ε ϛ′ κ′γ]′ ρλ′[η′]	of 5, $\overline{6}$ $\overline{23}$ $\overline{138}$

P. Mich. III.145, I,ii [49, p. 36]:

των ιβ d η′ κϑ′ σλ′β′	of 12, $\overline{4}$ $\overline{8}$ $\overline{29}$ $\overline{232}$
[των] ιγ γ′ ι′ε′ [κ′ϑ′ π′]ζ′ υ[λ′ε′]	of 13, $\overline{3}$ $\overline{15}$ $\overline{29}$ $\overline{87}$ $\overline{435}$
[των] ιδ d ε′ [ν′]η′ ρις′ πμ′ε′	of 14, $\overline{4}$ $\overline{5}$ $\overline{58}$ $\overline{116}$ $\overline{145}$
[των] ιε ∠ ν̣′ η′	of 15, $\overline{2}$ $\overline{58}$
[των ι] ς̣ [∠ κ′] ϑ̣′ ν′η′	of 16, $\overline{29}$ $\overline{58}$
[των ιζ ∠ ι′β′] τ′μ′η′	of 17, $\overline{2}$ $\overline{12}$ $\overline{348}$

P. Mich. III.146, Inv. 621, fourth century AD, from the Fayum [49, pp. 52–58], edited and described previously by Robbins [35]. Robbins [35, p. 328]:

> The papyrus numbered 621 in the recently acquired collection of the University of Michigan, since it contains tables of fractions, adds another to the rather brief list of documents bearing upon logistic, which, as the science of calculating, was clearly distinguished by the ancients from arithmetic, the science of numbers as such. It is one of the longest rolls of the collection, and of peculiar shape, since it is almost exactly 3 feet 6 inches long and, on the average, only $3\frac{5}{8}$ inches (92 mm.) high. It came originally from the Fayum, and dates from approximately the fourth century A.D.; there are none

but paleographic clues to its age. It is clearly written, in a uniform hand, on the recto only, the verso remaining blank. There are 19 narrow columns, of which the first is mutilated and the last, save for the heading, left blank. It may be inferred that the beginning of the tables has been lost, and that the scribe, who undoubtedly was copying, not actually calculating, left off without completing his task.

Papyrus 621 contains a list of the fractions of numbers, beginning with the last part of the sevenths and continuing with eighths, ninths, and so on, through the eighteenths; the heading "nineteenths" appears, but nothing is written under it. Through the tenths, the fractions for the numbers 1, 2, 3, 4, ... 10, 20, 30, 40 .... 100, 200, 300, 400 .... 1,000, 2,000, 3,000, 4,000 .... 10,000 are given, but thereafter only for the successive numbers up to the denominator of the fraction itself. The results are invariably expressed as sums of fractions with unit numerators, with the single exception of $\frac{2}{3}$, which in both Egyptian and Greek logistic seems to have ranked with the unit fractions and had a special sign. In each table the second entry consists of the denominiator of the fraction followed by the number which multiplied thereby gives 6,000; this was included, doubtless, because 6,000 drachmas make a talent.

A talent is a unit of weight, typically used with silver; a talent of silver has the weight of around 26 kg.

Sevenths: Col. i; eighths: Col. i–iv; ninths: Col. iv–vii; tenths: Col. vii–ix; elevenths: Col. x; twelfths: Col. xi; thirteenths: Col. xii; fourteenths: Col. xiii; fifteenths: Col. xiv–xv; sixteenths: Col. xv–xvi; seventeenths: Col. xvi–xvii; eighteenths: Col. xvii–xviii; nineteenths: Col. xix, intact but blank except for the heading εννεακαιδεκατα.

P. Mich. III.146, sevenths, Col. i [49, p. 54]:

[των Α] ρ′μ′β′∠′γ′μ′β′	of 1000, 142 $\overline{2}$ $\overline{3}$ $\overline{42}$
[των Β] σ′π′εΓ′κ′α′	of 2000, 285 $\overline{\overline{3}}$ $\overline{21}$
[των Γ] υ′κ′η′∠′ι′δ′	of 3000, 428 $\overline{2}$ $\overline{14}$
[των Δ] φ′ο′α′γ′ι′ε′λ′ε′	of 4000, 571 $\overline{3}$ $\overline{15}$ $\overline{35}$
[των Ε] ψ′ι′δ′δ′κ′η′	of 5000, 714 $\overline{4}$ $\overline{28}$
[των Ϛ] ω′ν′ζ′ζ′	of 6000, 857 $\overline{7}$
[των Ζ] Α′	of 7000, 1000
[των Η] Α′ρ′μ′β′∠′γ′μ′β′	of 8000, 1142 $\overline{2}$ $\overline{3}$ $\overline{42}$
[των Θ] Α′σ′π′εΓ′κ′α′	of 9000, 1285 $\overline{\overline{3}}$ $\overline{21}$
[των $\overset{\alpha}{\mu}$] Α′υ′κ′η′∠′ι′δ′	of 10000, 1428 $\overline{2}$ $\overline{14}$

MPER XV.167 = P. Vindob. G 24550, Soknopaiou Nesos, Fayum, second century AD [21, pp. 160–161], table of sevenths, Col. I–II: the table has further entries for 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10000. These entries except for 4000 are the same as the entries in P. Mich. III.146; the entry for 4000 in P. Mich. III.146 is φ′ο′α′γ′ι′ε′λ′ε′, 571 $\overline{3}$ $\overline{15}$ $\overline{35}$, which is right, and the entry in MPER XV.167 is [φο]αγʹιαλε, 571 $\overline{3}\ \overline{11}\ \overline{35}$, which is wrong. However, the editors transcribe the entry in MPER XV.167 as [φο]α̣γʹια̣λε. α̣ indicates that the reading α is uncertain.

[ἕβ]δομ[ον]	
[τῆς] α [μ]ιᾶς τὸ $\overline{\zeta}$ $\overline{\zeta}$	of 1, $\overline{7}$
τῶν β dκη	of 2, $\overline{4}\ \overline{28}$
τῶν γ γʹιαʹλε	of 3, $\overline{3}\ \overline{11}\ \overline{35}$ [!]
[τῶν δ] ∠ιδ	of 4, $\overline{2}\ \overline{14}$
[τῶν ε] Γκα	of 5, $\overline{\overline{3}}\ \overline{21}$
[τῶν] ϛ ∠γʹμβ	of 6, $\overline{2}\ \overline{3}\ \overline{42}$
τῶν ζ α	of 7, 1
[τ]ῶ[ν] η αζ[ʹ]	of 8, 1 $\overline{7}$
τῶν ϑ αdʹκη	of 9, 1 $\overline{4}\ \overline{28}$
τῶν ι [αγʹ]ιαλε	of 10, 1 $\overline{3}\ \overline{11}\ \overline{35}$ [!]
[τῶ]ν κ β∠γʹμβ	of 20, 2 $\overline{2}\ \overline{3}\ \overline{42}$
τῶν μ ε[Γ]ʹ$\overline{\kappa}$ $\overline{\alpha}$	of 40, 5 $\overline{\overline{3}}\ \overline{21}$
τ[ῶ]ν [ν ζζʹ]	of 50, 7 $\overline{7}$
[τῶν ξ η∠ι]δ	of 60, 8 $\overline{2}\ \overline{14}$
[τῶν ο ι]	of 70, 10
[τῶν π ιαι]αʹλε	of 80, 11 $\overline{11}\ \overline{35}$ [!]
[τῶν] ϙ [ιβ∠γʹ]μβ	of 90, 12 $\overline{2}\ \overline{3}\ \overline{42}$
[τῶν ρ ιδdʹκη]ʹ	of 100, 14 $\overline{4}\ \overline{28}$
[τῶν σ κη∠ι]δʹ	of 200, 28 $\overline{2}\ \overline{14}$
[τῶν τ μβ∠γ]ʹμβ	of 300, 42 $\overline{2}\ \overline{3}\ \overline{42}$
[τῶν υ νζζ]ʹ	of 400, 57 $\overline{7}$
[τῶν φ οαγʹια]λε	of 500, 71 $\overline{3}\ \overline{11}\ \overline{35}$ [!]
[τῶν χ πεΓ]κα	of 600, 85 $\overline{\overline{3}}\ \overline{21}$
[τῶν ψ ρ]	of 700, 100
[τῶν ω ρ[ιδ]dκη	of 800, 114 $\overline{4}\ \overline{28}$
[τῶν] ϡ ρκη∠ιδ	of 900, 128 $\overline{2}\ \overline{14}$

τῶν γ: $\frac{3}{7} = \overline{3}\ \overline{11}\ \overline{231}$ and $\frac{3}{7} = \overline{3}\ \overline{15}\ \overline{235}$.

τῶν ι: $\frac{10}{7} = 1 + \frac{3}{7}$.

[**τῶν π**: $\frac{80}{7} = \frac{70}{7} + \frac{10}{7} = 10 + \frac{10}{7}$.

[**τῶν φ**:

[**των A**]: $7 \cdot 100 = 700$, $7 \cdot 40 = 280$, $7 \cdot 2 = 14$. $7 \cdot 142 = 994$. $7 \cdot (142 + x) = 1000$. $994 + 7x = 1000$. $7x = 6$.

$$x = \frac{6}{7} = \overline{2}\ \overline{3}\ \overline{42}.$$

$$\frac{1000}{7} = 142 + x = 142\ \overline{2}\ \overline{3}\ \overline{42}.$$

Thus the seventh part of A is ρʹμʹβʹ ∠ʹ γʹμʹβʹ.

[**των B**]:

$$\frac{2000}{7} = 284\ 1\ \overline{\overline{3}}\ \overline{21} = 285\ \overline{\overline{3}}\ \overline{21}.$$

Thus the seventh part of B is σ′π′ε′ Γ′ϰ′α′.

[των Γ]: $7\cdot 400 = 2800$, $7\cdot 20 = 140$, $7\cdot 8 = 56$. $7\cdot 428 = 2996$. $7\cdot(428+x) = 3000$. $7x = 4$.

$$x = \frac{4}{7} = \frac{8}{14} = \frac{1}{14} + \frac{7}{14} = \overline{2}\ \overline{14}.$$

$$\frac{3000}{7} = 428 + x = 428\ \overline{2}\ \overline{14}.$$

Thus the seventh part of Γ is υ′ϰ′η′∠′ι′δ′.

[των Δ]: Using $\frac{2000}{7}$ and $\frac{2}{21} = \overline{14}\ \overline{42}$ (RMP Recto):

$$\frac{4000}{7} = 570\ 1\ \overline{3}\ \overline{14}\ \overline{42} = 571\ \overline{3}\ \overline{14}\ \overline{42}.$$

Thus the seventh part of Δ is φ′ο′α′γ′ι′δ′μ′β′.

On the one hand, the entry in col. i is φ′ο′α′γ′ι′ε′λ′ε′, $571\ \overline{3}\ \overline{15}\ \overline{35}$, different than the expression just caclculated; on the other hand, the entry in the table of sevenths of the Akhmim Mathematical Papyrus, col. 5 [2, p. 27] is ΦΟΑγ′ιδ′μβ′. Using $\overline{14} = \overline{15}\ \overline{210}$ and $\overline{35} = \overline{42}\ \overline{210}$,

$$\begin{aligned}\frac{4000}{7} &= 571\ \overline{3}\ \overline{14}\ \overline{42}\\ &= 571\ \overline{3}\ (\overline{15}\ \overline{210})\ \overline{42}\\ &= 571\ \overline{3}\ \overline{15}\ (\overline{42}\ \overline{210})\\ &= 571\ \overline{3}\ \overline{15}\ \overline{35}.\end{aligned}$$

Thus the seventh part of Δ is φ′ο′α′γ′ι′ε′λ′ε′.

[των Ε]: $7\cdot 700 = 4900$, $7\cdot 10 = 70$, $7\cdot 4 = 28$. $7\cdot 714 = 4998$. $7\cdot(714+x) = 5000$. $7x = 2$.

$$x = \frac{2}{7} = \overline{4}\ \overline{28}.$$

$$\frac{5000}{7} = 714\ \overline{4}\ \overline{28}.$$

Thus the seventh part of E is ψ′ι′δ′δ′ϰ′η′.

[των Ϛ]: Using $\frac{3000}{7}$,

$$\frac{6000}{7} = 856\ 1\ \overline{7} = 857\ \overline{7}.$$

Thus the seventh part of Ϛ is ω′ν′ζ′ζ′.

[των Z]: The seventh part of Z is A′.

[των H]: Using the expression worked out for $\frac{4000}{7}$ and $\overline{7}\ \overline{21} = \overline{6}\ \overline{42}$,

$$\frac{8000}{7} = 1142\ \overline{\overline{3}}\ \overline{7}\ \overline{21} = 1142\ \overline{\overline{3}}\ \overline{6}\ \overline{42} = 1142\ \overline{2}\ \overline{3}\ \overline{42}.$$

Thus the seventh part of H is A′ρ′μ′β′∠′γ′μ′β′.

[**των Θ**]: Using $\frac{3000}{7} = 428\ \overline{2}\ \overline{14}$ and $\frac{3}{14} = \overline{5}\ \overline{70}$,

$$\frac{9000}{7} = 1284\ 1\ \overline{2}\ \overline{5}\ \overline{70} = 1285\ \overline{2}\ \overline{5}\ \overline{70}.$$

Thus the seventh part of Θ is Α′σ′π′ε′∠′ε′ο′. Calculated differently, $7{\cdot}1000 = 7000$, $7 \cdot 200 = 1400$, $7 \cdot 80 = 560$, $7 \cdot 5 = 35$. $7 \cdot 1285 = 8995$. $7 \cdot (1285 + x) = 9000$. $7x = 5$.

$$x = \frac{5}{7} = \overline{2}\ \overline{5}\ \overline{70}.$$

$$\frac{9000}{7} = 1285 + x = 1285\ \overline{2}\ \overline{5}\ \overline{70},$$

which is the same expression found above for $\frac{9000}{7}$. On the other hand, the entry in the table is Α′σ′π′εΓ′ϰ′α′, i.e. $1285\ \overline{\overline{3}}\ \overline{21}$, which is also the entry in the table of sevenths of the Akhmim Mathematical Papyrus, col. 5 [2, p. 27]. Using $\overline{2}\ \overline{5} = \overline{\overline{3}}\ \overline{30}$,

$$\overline{2}\ \overline{5}\ \overline{70} = \overline{\overline{3}}\ \overline{30}\ \overline{70}.$$

$\overline{6}\ \overline{30} = \overline{5}$. $\overline{84}\ \overline{420} = \overline{70}$.

$$\overline{\overline{3}}\ \overline{30}\ \overline{70} = \overline{\overline{3}}\ \overline{30}\ \overline{84}\ \overline{420} = \overline{\overline{3}}\ \overline{84}\ (\overline{30}\ \overline{420}) = \overline{\overline{3}}\ \overline{84}\ \overline{28} = \overline{\overline{3}}\ \overline{28}\ \overline{84} = \overline{\overline{3}}\ \overline{21}.$$

Then

$$\frac{9000}{7} = 1285\ \overline{\overline{3}}\ \overline{21}.$$

Thus the seventh part of Θ is Α′σ′π′εΓ′ϰ′α′.

[**των** $\overset{\alpha}{\mu}$]: $7{\cdot}1000 = 7000$, $7{\cdot}400 = 2800$, $7{\cdot}20 = 140$, $7{\cdot}8 = 56$. $7{\cdot}1428 = 9996$. $7 \cdot (1428 + x) = 10000$. $7x = 4$.

$$x = \frac{4}{7} = \overline{2}\ \overline{14}.$$

$$\frac{10000}{7} = 1428 + x = 1428\ \overline{2}\ \overline{14}.$$

Thus the seventh part of $\overset{\alpha}{\mu}$ is Α′υ′ϰ′η′∠′ι′δ′.

P. Mich. III.146, seventeenths, επταϰαιδεϰατα, Col. xvi–xvii [49, p. 57]:

της α το ιζιζ	of 1, $\overline{17}$
το [ι]ζ τ′ν′β′∠′γ′ι′ζ′λ′δ′ν′α′	of 6000, 352 $\overline{2}\ \overline{3}\ \overline{17}\ \overline{34}\ \overline{51}$
τω[ν] β ι′β′ν′α′ξ′η′	of 2, $\overline{12}\ \overline{51}\ \overline{68}$
τω[ν γ] ι′β′ι′ζ′ν′α′ξ′η′	of 3, $\overline{12}\ \overline{17}\ \overline{51}\ \overline{68}$
τω[ν δ] ι′β′ι′ε′ι′ζ′λ′δ′ν′α′	of 4, $\overline{12}\ \overline{15}\ \overline{17}\ \overline{34}\ \overline{51}$
τ[ων] ε δ′λ′δ′ξ′η′	of 5, $\overline{4}\ \overline{34}\ \overline{68}$
των ϛ′ γ′ν′α′	of 6, $\overline{3}\ \overline{51}$
των ζ γ′ι′ζ′ν′α′	of 7, $\overline{3}\ \overline{17}\ \overline{51}$
των η γ′ι′ε′ι′ζ′π′ε′	of 8, $\overline{3}\ \overline{15}\ \overline{17}\ \overline{85}$
των ϑ ∠′λ′δ′	of 9, $\overline{2}\ \overline{34}$
των ι ∠′ι′ζ′λ′δ′	of 10, $\overline{2}\ \overline{17}\ \overline{34}$
των ια ∠′ι′β′ι′ζ′λ′δ′ν′α′	of 11, $\overline{2}\ \overline{12}\ \overline{17}\ \overline{34}\ \overline{51}$ [!]
των ιβ ∠′ι′β′ι′ζ′λ′δ′ν′α′ξ′η′	of 12, $\overline{2}\ \overline{12}\ \overline{17}\ \overline{34}\ \overline{51}\ \overline{68}$
των ιγ ∠′δ′ξ′η′	of 13, $\overline{2}\ \overline{4}\ \overline{68}$
των ιδ ∠′δ′ι′ζ′ξ′η′	of 14, $\overline{2}\ \overline{4}\ \overline{17}\ \overline{68}$
των ιε ∠′δ′ι′ζ′λ′δ′ξ′η′	of 15, $\overline{2}\ \overline{17}\ \overline{34}\ \overline{68}$ [!]
των ιϛ ∠′γ′ι′ζ′λ′δ′ν′α′	of 16, $\overline{2}\ \overline{3}\ \overline{17}\ \overline{34}\ \overline{51}$
των ιζ α	of 17, 1

της α: The seventeenth part of 1 is $\overline{17}$, i.e. a is ιζ. The entry in the table is ιζιζ. Robbins [35, p. 329]:

> There is no way of distinguishing fractions from integers; for example, γ′ can mean either 3 or $\frac{1}{3}$, and γ′γ′ occurs in the sense $3\frac{1}{3}$. The only exception is that in the first entry in each table the doubling of the letter shows that it denotes a fraction.

το [ι]ζ: $17 \cdot 300 = 5100$, $17 \cdot 50 = 850$, $17 \cdot 2 = 34$. $17 \cdot 352 = 5984$. $17 \cdot (352 + x) = 6000$. $17x = 16$. $x = \frac{16}{17}$.

RMP $\frac{2}{n}$ table, entry for $\frac{2}{17}$ is the following [11, pp. 123–124].

> **Call 2** out of 17 [i.e., Get 2 by operating on 17].
> **1/12** [of 17 is] 1 1/3 1/12, **1/51** [of 17 is] 1/3, **1/68** [of 17 is] 1/4.
> **Procedure:**

	1	17		
	2/3	11 1/3		
	1/3	5 2/3	\	1 17
	1/6	2 1/2 1/3	\	2 34
\	1/12	1 1/4 1/6 [Total:]		3 51 1/3

> Remainder 1/3 1/4

This entry is explained by Clagett [11, p. 34]; cf. Chace [10, pp. 16–117]. $\frac{1}{12} \cdot 17 = 1\,\overline{4}\,\overline{6}$. $\frac{1}{12} \cdot 17 + R = 2$. $\frac{1}{12} + R \cdot \frac{1}{17} = \frac{2}{17}$.

$$R = 2 - \frac{1}{12} \cdot 17 = 2 - (1\,\overline{4}\,\overline{6}) = 1 - (\overline{4}\,\overline{6}) = (\overline{2} - \overline{4}) + (\overline{2} - \overline{6}) = \overline{4} + \overline{3}.$$

$R = \overline{3}\,\overline{4}$ is the Remainder ($\underline{\text{d}}$3t). Therefore

$$\frac{2}{17} = \frac{1}{12} + R \cdot \frac{1}{17} = \overline{12} + \overline{3} \cdot \overline{17} + \overline{4} \cdot \overline{17} = \overline{12}\,\overline{51}\,\overline{68}.$$

Using $\frac{2}{17} = \overline{12}\,\overline{51}\,\overline{68}$ and $\frac{2}{51} = \overline{34}\,\overline{102}$ from the RMP $\frac{2}{n}$ table [11, p. 128],

$$\frac{4}{17} = \overline{6} + \frac{2}{51} + \overline{34} = \overline{6}\;(\overline{34}\,\overline{102})\;\overline{34} = \overline{6}\;(\overline{34}\,\overline{34})\;\overline{102} = \overline{6}\,\overline{17}\,\overline{102}.$$

Using $\frac{4}{17} = \overline{6}\,\overline{17}\,\overline{102}$ $\frac{2}{17} = \overline{12}\,\overline{51}\,\overline{68}$, and $\frac{2}{51} = \overline{34}\,\overline{102}$,

$$\begin{aligned}
\frac{8}{17} &= \overline{3} + \frac{2}{17} + \overline{51} \\
&= \overline{3}\;(\overline{12}\,\overline{51}\,\overline{68})\;\overline{51} \\
&= \overline{3}\,\overline{12}\;(\overline{51}\,\overline{51})\;\overline{68} \\
&= \overline{3} + \overline{12} + \frac{2}{51} + \overline{68} \\
&= \overline{3} + \overline{12} + (\overline{34} + \overline{102}) + \overline{68} \\
&= \overline{3}\,\overline{12}\,\overline{34}\,\overline{68}\,\overline{102}.
\end{aligned}$$

Using $\frac{8}{17} = \overline{3}\,\overline{12}\,\overline{34}\,\overline{68}\,\overline{102}$ and $\frac{2}{3} = \overline{2}\,\overline{6}$,

$$\frac{16}{17} = (\overline{2}\,\overline{6})\;\overline{6}\,\overline{17}\,\overline{34}\,\overline{51} = \overline{2}\,\overline{3}\,\overline{17}\,\overline{34}\,\overline{51}.$$

Therefore, with $17 \cdot (352 + x) = 6000$, for which $x = \frac{16}{17}$,

$$\frac{6000}{17} = 352 + x = 352 + \frac{16}{17} = 352\;\overline{2}\,\overline{3}\,\overline{17}\,\overline{34}\,\overline{51}.$$

Therefore the seventeenth part of ἈϡΤ is τ′ν′β′∠′γ′ι′ζ′λ′δ′ν′α′.

τω[ν] β: Using $\frac{2}{17} = \overline{12}\,\overline{51}\,\overline{68}$ from the RMP $\frac{2}{n}$ table, the seventeenth part of β′ is ι′β′ν′α′ξ′η′.

On the other hand, using

$$\frac{a}{bc} = \frac{1}{c \cdot \frac{b+c}{a}} + \frac{1}{b \cdot \frac{b+c}{a}}$$

with $a = 2, b = 17, c = 1$,

$$\frac{2}{17} = \frac{1}{1 \cdot 9} + \frac{1}{17 \cdot 9} = \overline{9}\,\overline{153}.$$

τω[ν γ]: Using $\frac{2}{17} = \overline{12}\,\overline{51}\,\overline{68}$,

$$\frac{3}{17} = \overline{17} + \frac{2}{17} = \overline{17}\;(\overline{12}\,\overline{51}\,\overline{68}) = \overline{12}\,\overline{17}\,\overline{51}\,\overline{68}.$$

Therefore the seventeenth part of γ is ι′β′ι′ζ′ν′α′ξ′η′.

On the other hand, using

$$\frac{a}{bc} = \frac{1}{c \cdot \frac{b+c}{a}} + \frac{1}{b \cdot \frac{b+c}{a}}$$

with $a = 3, b = 17, c = 1$,

$$\frac{3}{17} = \frac{1}{1 \cdot 6} + \frac{17 \cdot 6}{=} \overline{6}\ \overline{102}.$$

τω[ν δ]: Using $\frac{3}{17} = \overline{6}\ \overline{102}$,

$$\frac{4}{17} = \frac{3}{17} + \frac{1}{17} = (\overline{6}\ \overline{102})\ \overline{17} = \overline{6}\ \overline{17}\ \overline{102}.$$

Thus the seventeenth part of δ is ϛ′ι′ζ′ρ′β′. The entry in the table is ι′β′ι′ε′ι′ζ′λ′δ′ν′α′, $\overline{12}\ \overline{15}\ \overline{17}\ \overline{34}\ \overline{51}$, which is wrong; the difference of this and $\frac{4}{17}$ is $\frac{23}{1020}$ not 0. However in the table of seventeenths in the Akhmim Mathematical Papyrus, col. 11 [2, p. 30], the entry for the seventeenth part of Δ is ιβ′ιε′ιζ′ξη′πε′, $\overline{12}\ \overline{15}\ \overline{17}\ \overline{68}\ \overline{85}$.

We calculate $\frac{2}{17}$ differently. Rather than using $\frac{2}{17} = \overline{12}\ \overline{51}\ \overline{68}$ from the RMP $\frac{2}{n}$ table, we use $\frac{2}{17} = \overline{17}\ \overline{24}\ \overline{102}\ \overline{136}$. Using $\frac{2}{85} = \overline{51}\ \overline{255}$ from the RMP $\frac{2}{n}$ table [11, p. 131],

$$\begin{aligned}
\frac{4}{17} &= \frac{2}{17} + \overline{12} + \overline{51} + \overline{68} \\
&= (\overline{17}\ \overline{24}\ \overline{102}\ \overline{136})\ \overline{12}\ \overline{51}\ \overline{68} \\
&= \overline{12}\ \overline{17}\ \overline{24}\ (\overline{51}\ \overline{102})\ \overline{68}\ \overline{136} \\
&= \overline{12}\ \overline{17}\ \overline{24}\ \overline{34}\ \overline{68}\ \overline{136} \\
&= \overline{12}\ \overline{17}\ \overline{24}\ \overline{68}\ (\overline{34}\ \overline{136}) \\
&= \overline{12}\ \overline{17}\ \overline{24}\ \overline{68}\ (\overline{40}\ \overline{85}) \\
&= \overline{12}\ \overline{17}\ (\overline{24}\ \overline{40})\ \overline{68}\ \overline{85} \\
&= \overline{12}\ \overline{15}\ \overline{17}\ \overline{68}\ \overline{85}.
\end{aligned}$$

Therefore, the seventeenth part of δ is ιβ ιε ιζ ξη πε, which is the entry in the Akhmim Mathematical Papyrus, col. 11 [2, p. 30].

τ[ων] ε: Using $\frac{2}{17} = \overline{12}\ \overline{51}\ \overline{68}$ and $\frac{3}{17} = \overline{12}\ \overline{17}\ \overline{51}\ \overline{68}$, and $\frac{2}{17} = \overline{12}\ \overline{51}\ \overline{68}$ and $\frac{2}{51} = \overline{34}\ \overline{102}$ from the RMP $\frac{2}{n}$ table [11, pp. 123, 128],

$$\begin{aligned}
\frac{5}{17} &= (\overline{12}\ \overline{12})\ \overline{17}\ (\overline{51}\ \overline{51})\ (\overline{68}\ \overline{68}) \\
&= \overline{6}\ \overline{17}\ \frac{2}{51}\ \overline{34} \\
&= \overline{6}\ \overline{17}\ (\overline{34}\ \overline{102})\ \overline{34} \\
&= \overline{6}\ \overline{17}\ \overline{17}\ \overline{102} \\
&= \overline{6}\ \frac{2}{17}\ \overline{102} \\
&= \overline{6}\ (\overline{12}\ \overline{51}\ \overline{68})\ \overline{102} \\
&= (\overline{6}\ \overline{12})\ (\overline{51}\ \overline{102})\ \overline{68} \\
&= \overline{4}\ \overline{34}\ \overline{68}.
\end{aligned}$$

Therefore, the seventeenth part of ε is δ′λ′δ′ξ′η′.

των ϛ′: Using $\frac{3}{17} = \overline{12}\;\overline{17}\;\overline{51}\;\overline{68}$,

$$\begin{aligned}\frac{6}{17} &= \overline{6}\;\frac{2}{17}\;\frac{2}{51}\;\overline{34}\\ &= \overline{6}\;(\overline{12}\;\overline{51}\;\overline{68})\;(\overline{34}\;\overline{102})\;\overline{34}\\ &= \overline{6}\;\overline{12}\;\overline{51}\;(\overline{17}\;\overline{68}\;\overline{102})\\ &= \overline{6}\;\overline{12}\;\overline{51}\;\overline{12}\\ &= \overline{6}\;(\overline{12}\;\overline{12})\;\overline{51}\\ &= \overline{6}\;\overline{6}\;\overline{51}\\ &= \overline{3}\;\overline{51}.\end{aligned}$$

On the other hand, using

$$\frac{a}{bc} = \frac{1}{c\cdot\frac{b+c}{a}} + \frac{1}{b\cdot\frac{b+c}{a}}$$

with $a = 6, b = 17, c = 1$,

$$\frac{6}{17} = \frac{1}{1\cdot\frac{18}{6}} + \frac{1}{17\cdot\frac{18}{6}} = \frac{1}{3} + \frac{1}{51}.$$

Thus,

$$\frac{6}{17} = \overline{3}\;\overline{51}.$$

Therefore the seventeenth part of ϛ is γ′ν′α′.

των ζ: Using $\frac{6}{17} = \overline{3}\;\overline{51}$,

$$\frac{7}{17} = \frac{6}{17} + \frac{1}{17} = (\overline{3};\overline{51})\;\overline{17} = \overline{3}\;\overline{17}\;\overline{51}.$$

Therefore the seventeenth part of ζ is γ′ι′ζ′ν′α′.

των η: Using $\frac{7}{17} = \overline{3}\;\overline{17}\;\overline{51}$, and $\overline{17}\;\overline{51} = \overline{15}\;\overline{85}$,

$$\begin{aligned}\frac{8}{17} = \frac{7}{17} + \frac{1}{17} &= (\overline{3}\;\overline{17}\;\overline{51})\overline{17}\\ &= \overline{3}\;(\overline{17}\;\overline{51})\;\overline{17}\\ &= \overline{3}\;(\overline{15}\;\overline{85})\;\overline{17}\\ &= \overline{3}\;\overline{15}\;\overline{17}\;\overline{85}.\end{aligned}$$

Therefore the seventeenth part of η is γ′ι′ε′ι′ζ′π′ε′.

των ϑ: Using the expressions worked out for $\frac{3}{17}$ and $\frac{6}{17}$,

$$\begin{aligned}\frac{9}{17} &= \frac{6}{17} + \frac{3}{17}\\ &= (\overline{3}\;\overline{51})\;(\overline{6}\;\overline{102})\\ &= (\overline{3}\;\overline{6})\;(\overline{51}\;\overline{102})\\ &= \overline{2}\;\overline{34}.\end{aligned}$$

Therefore the seventeenth part of ϑ is ∠′λ′δ′.

των ι: Using the expression worked out for $\frac{2}{17}$,

$$\frac{10}{17} = \frac{9}{17} + \frac{1}{17} = (\overline{2}\;\overline{34})\;\overline{17} = \overline{2}\;\overline{17}\;\overline{34}.$$

Therefore the seventeenth part of ι is ∠′ι′ζ′λ′δ′

των ια: Using the expression worked out for $\frac{10}{17}$, and $\frac{2}{17} = \overline{12}\;\overline{51}\;\overline{68}$ from the RMP $\frac{2}{n}$ table [11, p. 128],

$$\begin{aligned}\frac{11}{17} &= \frac{10}{17} + \frac{1}{17} \\ &= (\overline{2}\;\overline{17}\;\overline{34})\;\overline{17} \\ &= \overline{2}\;(\overline{17}\;\overline{17})\;\overline{34} \\ &= \overline{2}\;(\overline{12}\;\overline{51}\;\overline{68})\;\overline{34} \\ &= \overline{2}\;\overline{12}\;\overline{34}\;\overline{51}\;\overline{68}.\end{aligned}$$

Therefore, the seventeenth part of ια is <ιβ′λδ′να′ξη′.

The entry in the table is ∠′ι′β′ι′ζ′λ′δ′ν′α′, $\overline{2}\;\overline{12}\;\overline{17}\;\overline{34}\;\overline{51}$, which is wrong. On the other hand, in the table of seventeenths in the Akhmim Mathematical Papyrus, col. 11 [2, p. 30], the entry for the seventeenth part of IA is <ιβ′λδ′να′ξη′, $\overline{2}\;\overline{12}\;\overline{34}\;\overline{51}\;\overline{68}$.

των ιβ: Using the expression worked out for $\frac{11}{17}$,

$$\frac{12}{17} = \frac{11}{17} + \frac{1}{17} = (\overline{2}\;\overline{12}\;\overline{34}\;\overline{51}\;\overline{68})\;\overline{17} = \overline{2}\;\overline{12}\;\overline{17}\;\overline{34}\;\overline{51}\;\overline{68}.$$

Therefore the seventeenth part of ιβ is ∠′ι′β′ι′ζ′λ′δ′ν′α′ξ′η′.

των ιγ: Using the expression worked out for $\frac{12}{17}$, the equality $\overline{17}\;\overline{34}\;\overline{102} = \overline{12}\;\overline{68}$, and $\frac{2}{17} = \overline{12}\;\overline{51}\;\overline{68}$ and $\frac{2}{51} = \overline{34}\;\overline{102}$ from the RMP $\frac{2}{n}$ table [11, pp. 123, 128],

$$\begin{aligned}\frac{13}{17} &= \frac{12}{17} + \frac{1}{17} \\ &= (\overline{2}\;\overline{12}\;\overline{17}\;\overline{34}\;\overline{51}\;\overline{68})\;\overline{17} \\ &= \overline{2}\;\overline{12}\;(\overline{17}\;\overline{17})\;\overline{34}\;\overline{51}\;\overline{68} \\ &= \overline{2}\;\overline{12}\;(\overline{12}\;\overline{51}\;\overline{68})\;\overline{34}\;\overline{51}\;\overline{68} \\ &= \overline{2}\;(\overline{12}\;\overline{12})\;(\overline{51}\;\overline{51})\;\overline{34}\;(\overline{68}\;\overline{68}) \\ &= \overline{2}\;\overline{6}\;(\overline{34}\;\overline{102})\;\overline{34}\;\overline{34} \\ &= \overline{2}\;\overline{6}\;(\overline{34}\;\overline{102})\;\overline{17} \\ &= \overline{2}\;\overline{6}\;(\overline{17}\;\overline{34}\;\overline{102}) \\ &= \overline{2}\;\overline{6}\;(\overline{12}\;\overline{68}) \\ &= \overline{2}\;(\overline{6}\;\overline{12})\;\overline{68} \\ &= \overline{2}\;\overline{4}\;\overline{68}.\end{aligned}$$

Therefore the seventeenth part of ιγ is ∠′δ′ξ′η′

των ιδ: Using the expression worked out for $\frac{13}{17}$,

$$\frac{14}{17} = \frac{13}{17} + \frac{1}{17} = (\overline{2}\ \overline{4}\ \overline{68})\ \overline{17} = \overline{2}\ \overline{4}\ \overline{17}\ \overline{68}.$$

Therefore the seventeenth part of ιδ is ∠′δ′ι′ζ′ξ′η′.

των ιε: Using the expression worked out for $\frac{14}{17}$, and $\frac{2}{17} = \overline{12}\ \overline{51}\ \overline{68}$ from the RMP $\frac{2}{n}$ [11, p. 128],

$$\begin{aligned} \frac{15}{17} &= \frac{14}{17} + \frac{1}{17} \\ &= (\overline{2}\ \overline{4}\ \overline{17}\ \overline{68})\ \overline{17} \\ &= \overline{2}\ \overline{4}\ (\overline{17}\ \overline{17})\ \overline{68} \\ &= \overline{2}\ \overline{4}\ (\overline{12}\ \overline{51}\ \overline{68})\ \overline{68} \\ &= \overline{2}\ \overline{4}\ \overline{12}\ \overline{51}\ \overline{34} \\ &= \overline{2}\ (\overline{4}\ \overline{12})\ \overline{34}\ \overline{51} \\ &= \overline{2}\ \overline{3}\ \overline{34}\ \overline{51}. \end{aligned}$$

Therefore the seventeenth part of ιε is <γ′λδ′να′.

The entry in the table is ∠′δ′ι′ζ′λ′δ′ξ′η′, $\overline{2}\ \overline{4}\ \overline{17}\ \overline{34}\ \overline{68}$, which is wrong. On the other hand, in the table of seventeenths in the Akhmim Mathematical Papyrus, col. 11 [2, p. 30], the entry for the seventeenth part of IE is <γ′λδ′να′.

των ιϛ: Using the expression worked out for $\frac{15}{17}$,

$$\frac{16}{17} = \frac{15}{17} + \frac{1}{17} = (\overline{2}\ \overline{3}\ \overline{34}\ \overline{51})\ \overline{17} = \overline{2}\ \overline{3}\ \overline{17}\ \overline{34}\ \overline{51}.$$

Therefore the seventeenth part of ιϛ is ∠′γ′ι′ζ′λ′δ′ν′α′.

των ιζ α: The seventeenth part of ιζ is α.

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