whether we take the logarithms of those quantities, as we have already done in the preceding section.

572. The equations which are most easily resolved, are those in which the unknown quantity does not exceed the first power, after the terms of the equation have been properly arranged; and these are called *simple equations*, or *equations of the first degree*. But if, after having reduced an equation, we find in it the square, or the second power, of the unknown quantity, it is called an *equation of the second degree*, which is more difficult to resolve. *Equations of the third degree* are those which contain the cube of the unknown quantity, and so on. We shall treat of all these in the present section.

CHAP. II.

Of the Resolution *of* Simple Equations, *or* Equations *of the* First Degree.

573. When the number sought, or the unknown quantity, is represented by the letter x, and the equation we have obtained is such, that one side contains only that x, and the other simply a known number, as, for example, $x = 25$, the value of x is already known. We must always endeavour, therefore, to arrive at such a form, however complicated the equation may be when first obtained: and, in the course of this section, the rules shall be given, and explained, which serve to facilitate these reductions.

574. Let us begin with the simplest cases, and suppose, first, that we have arrived at the equation $x + 9 = 16$. Here we see immediately that $x = 7$: and, in general, if we have found $x + a = b$, where a and b express any known numbers, we have only to subtract a from both sides, to obtain the equation $x = b - a$, which indicates the value of x.

575 If we have the equation $x - a = b$, we must add a to both sides, and shall obtain the value of $x = b + a$. We must proceed in the same manner, if the equation have this form, $x - a = a^2 + 1$: for we shall find immediately $x = a^2 + a + 1$.

In the equation $x - 8a = 20 - 6a$, we find

$$x = 20 - 6a + 8a, \text{ or } x = 20 + 2a.$$

And in this, $x + 6a = 20 + 3a$, we have

$$x = 20 + 3a - 6a, \text{ or } x = 20 - 3a.$$

576. If the original equation have this form, $x - a + b = c$, we may begin by adding a to both sides, which will give $x + b = c + a$; and then subtracting b from both sides, we shall find $x = c + a - b$. But we might also add $+ a - b$ at once to both sides; and thus obtain immediately $x = c + a - b$.

So likewise in the following examples:

If $x - 2a + 3b = 0$, we have $x = 2a - 3b$.

If $x - 3a + 2b = 25 + a + 2b$, we have $x = 25 + 4a$.

If $x - 9 + 6a = 25 + 2a$, we have $x = 34 - 4a$.

577. When the given equation has the form $ax = b$, we only divide the two sides by a, to obtain $x = \frac{b}{a}$. But if the equation has the form $ax + b - c = d$, we must first make the terms that accompany ax vanish, by adding to both sides $- b + c$; and then dividing the new equation $ax = d - b + c$ by a, we shall have $x = \frac{d-b+c}{a}$.

The same value of x would have been found by subtracting $+ b - c$ from the given equation; that is, we should have had, in the same form,

$ax = d - b + c$, and $x = \frac{d-b+c}{a}$. Hence,

If $2x + 5 = 17$, we have $2x = 12$, and $x = 6$.

If $3x - 8 = 7$, we have $3x = 15$, and $x = 5$.

If $4x - 5 - 3a = 15 + 9a$, we have $4x = 20 + 12a$, and consequently $x = 5 + 3a$.

578. When the first equation has the form $\frac{x}{a} = b$, we multiply both sides by a, in order to have $x = ab$.

But if it is $\frac{x}{a} + b - c = d$, we must first make $\frac{x}{a} = d - b + c$; after which we find

$$x = (d - b + c)\,a = ad - ab + ac.$$

Let $\frac{1}{2}x - 3 = 4$, then $\frac{1}{2}x = 7$, and $x = 14$.

Let $\frac{1}{3}x - 1 + 2a = 3 + a$, then $\frac{1}{3}x = 4 - a$, and $x = 12 - 3a$.

Let $\frac{x}{a-1} - 1 = a$, then $\frac{x}{a-1} = a + 1$, and $x = a^2 - 1$.

579. When we have arrived at such an equation as

$\frac{ax}{b} = c$, we first multiply by b, in order to have $ax = bc$, and then dividing by a, we find $x = \frac{bc}{a}$.

If $\frac{ax}{b} - c = d$, we begin by giving the equation this form $\frac{ax}{b} = d + c$; after which we obtain the value of $ax = bd + bc$, and then that of $x = \frac{bd+bc}{a}$.

Let $\frac{2}{3}x - 4 = 1$, then $\frac{2}{3}x = 5$, and $2x = 15$; whence $x = \frac{15}{2}, = 7\frac{1}{2}$.

If $\frac{3}{4}x + \frac{1}{2} = 5$, we have $\frac{3}{4}x = 5 - \frac{1}{2} = \frac{9}{2}$; whence $3x = 18$, and $x = 6$.

580. Let us now consider a case, which may frequently occur; that is, when two or more terms contain the letter x, either on one side of the equation, or on both.

If those terms are all on the same side, as in the equation $x + \frac{1}{2}x + 5 = 11$, we have $x + \frac{1}{2}x = 6$; or $3x = 12$; and lastly, $x = 4$.

Let $x + \frac{1}{2}x + \frac{1}{3}x = 44$, be an equation, in which the value of x is required. If we first multiply by 3, we have $4x + \frac{3}{2}x = 132$; then multiplying by 2, we have $11x = 264$; wherefore $x = 24$. We might have proceeded in a more concise manner, by beginning with the reduction of the three terms which contain x to the single term $\frac{11}{6}x$; and then dividing the equation $\frac{11}{6}x = 44$ by 11. This would have given $\frac{1}{6}x = 4$, and $x = 24$, as before.

Let $\frac{2}{3}x - \frac{3}{4}x + \frac{1}{2}x = 1$. We shall have, by reduction, $\frac{5}{12}x = 1$, and $x = 2\frac{2}{5}$.

And, generally, let $ax - bx + cx = d$; which is the same as $(a - b + c)x = d$, and, by division, we derive $x = \frac{d}{a-b+c}$.

581. When there are terms containing x on both sides of the equation, we begin by making such terms vanish from that side from which it is most easily expunged; that is to say, in which there are the fewest terms so involved.

If we have, for example, the equation $3x + 2 = x + 10$, we must first subtract x from both sides, which gives $2x + 2 = 10$; wherefore $2x = 8$, and $x = 4$.

Let $x + 4 = 20 - x$; here it is evident that $2x + 4 = 20$; and consequently $2x = 16$, and $x = 8$.

Let $x + 8 = 32 - 3x$, this gives us $4x + 8 = 32$; or $4x = 24$, whence $x = 6$.

Let $15 - x = 20 - 2x$, here we shall have

$$15 + x = 20, \text{ and } x = 5.$$

Let $1 + x = 5 - \frac{1}{2}x$; this becomes $1 + \frac{3}{2}x = 5$, or $\frac{3}{2}x = 4$; therefore $3x = 8$; and lastly, $x = \frac{8}{3} = 2\frac{2}{3}$.

If $\frac{1}{2} - \frac{1}{3}x = \frac{1}{3} - \frac{1}{4}x$, we must add $\frac{1}{3}x$, which gives $\frac{1}{2} = \frac{1}{3} + \frac{1}{12}x$; subtracting $\frac{1}{3}$, and transposing the terms, there remains $\frac{1}{12}x = \frac{1}{6}$; then multiplying by 12, we obtain $x=2$.

If $1\frac{1}{2} - \frac{2}{3}x = \frac{1}{4} + \frac{1}{2}x$, we add $\frac{2}{3}x$, which gives $1\frac{1}{2} = \frac{1}{4} + \frac{7}{6}x$; then subtracting $\frac{1}{4}$, and transposing, we have $\frac{7}{6}x = 1\frac{1}{4}$, whence we deduce $x = 1\frac{1}{14} = \frac{15}{14}$ by multiplying by 6 and dividing by 7.

582. If we have an equation in which the unknown number x is a denominator, we must make the fraction vanish by multiplying the whole equation by that denominator.

Suppose that we have found $\dfrac{100}{x} - 8 = 12$, then, adding 8, we have $\dfrac{100}{x} = 20$; and multiplying by x, it becomes $100 = 20x$; lastly, dividing by 20, we find $x = 5$.

Let now $\dfrac{5x+3}{x-1} = 7$; here multiplying by $x - 1$, we have $5x + 3 = 7x - 7$; and subtracting $5x$, there remains $3 = 2x - 7$; then adding 7, we have $2x = 10$; whence $x = 5$.

583. Sometimes, also, radical signs are found in equations of the first degree. For example: A number x, below 100, is required, such, that the square root of $100 - x$ may be equal to 8; or $\sqrt{(100 - x)} = 8$. The square of both sides will give $100 - x = 64$, and adding x, we have $100 = 64 + x$; whence we obtain $x = 100 - 64 = 36$.

Or, since $100 - x = 64$, we might have subtracted 100 from both sides; which would have given $-x = -36$; or, multiplying by -1, $x = 36$.

584. Lastly, the unknown number x is sometimes found as an exponent, of which we have already seen some examples; and, in this case, we must have recourse to logarithms.

Thus, when we have $2^x = 512$, we take the logarithms of both sides; whence we obtain $x\ log.\ 2 = log.\ 512$; and dividing by $log.\ 2$, we find $x = \dfrac{log.\ 512}{log.\ 2}$. The Tables then give, $x = \frac{2{\cdot}7092700}{0{\cdot}3010300} = \frac{270927}{30103}$, or $x = 9$.

Let $5 \times 3^{2x} - 100 = 305$; we add 100, which gives $5 \times 3^{2x} = 405$; dividing by 5, we have $3^{2x} = 81$; and taking the logarithms, $2x\ log.\ 3 = log.\ 81$, and dividing by $2\ log.\ 3$, we have $x = \frac{log.\ 81}{2 log.\ 3}$, or $x = \frac{log.\ 81}{log.\ 9}$; whence

$$x = \frac{1{\cdot}9084850}{0{\cdot}9542425} = \frac{19084850}{9542425} = 2.$$

QUESTIONS FOR PRACTICE.

1. If $x - 4 + 6 = 8$, then will $x = 6$.
2. If $4x - 8 = 3x + 20$, then will $x = 28$.
3. If $ax = ab - a$, then will $x = b - 1$.
4. If $2x + 4 = 16$, then will $x = 6$.
5. If $ax + 2ba = 3c^2$, then will $x = \frac{3c^2}{a} - 2b$.
6. If $\frac{x}{2} = 5 + 3$, then will $x = 16$.
7. If $\frac{2x}{3} - 2 = 6 + 4$, then will $2x - 6 = 18$.
8. If $a - \frac{b}{x} = c$, then will $x = \frac{b}{a-c}$.
9. If $5x - 15 = 2x + 6$, then will $x = 7$.
10. If $40 - 6x - 16 = 120 - 14x$, then will $x = 12$.
11. If $\frac{x}{2} - \frac{x}{3} + \frac{x}{4} = 10$, then will $x = 24$.
12. $\frac{x-3}{2} + \frac{x}{3} = 20 - \frac{x-19}{2}$, then will $x = 23\frac{1}{4}$.
13. If $\sqrt{\tfrac{2}{3}}x + 5 = 7$, then will $x = 6$.
14. If $x + \sqrt{(a^2 + x^2)} = \frac{2a^2}{\sqrt{(a^2+x^2)}}$, then will $x = a\sqrt{\tfrac{1}{3}}$.
15. If $3ax + \frac{a}{2} - 3 = bx - a$, then will $x = \frac{6-3a}{6a-2b}$.
16. If $\sqrt{(12 + x)} = 2 + \sqrt{x}$, then will $x = 4$.
17. If $y + \sqrt{(a^2 + y^2)} = \frac{2a^2}{(a^2+y^2)^{\frac{1}{2}}}$, then will $y = \frac{1}{3}a\sqrt{3}$.
18. If $\frac{y+1}{2} + \frac{y+2}{3} = 16 - \frac{y+3}{4}$, then will $y = 13$.

19. If $\surd x + \surd(a+x) = \frac{2a}{\surd(a+x)}$, then will $x = \frac{a}{3}$.

20. If $\surd(aa + xx) = \sqrt[4]{(b^4 + x^4)}$, then will $x = \frac{\surd(b4-a^4)}{2a^2}$.

21. If $y = \surd(a^2 + \surd(b^2 + x^2)) - a$, then will $x = \frac{bb}{4a} - a$.

22. If $\frac{128}{3x-4} = \frac{216}{5x-6}$, then will $x = 12$.

23. If $\frac{42x}{x-2} = \frac{35x}{x-3}$, then will $x = 8$.

24. If $\frac{45}{2x+3} = \frac{57}{4x-5}$, then will $x = 6$.

25. If $\frac{x^2-12}{3} = \frac{x^2-4}{4}$, then will $x = 6$.

26. If $615x - 7x^3 = 48x$, then will $x = 9$.

CHAP. III.

Of the Solution *of* Questions *relating to the preceding* Chapter.

585. *Question* 1. To divide 7 into two such parts that the greater may exceed the less by 3.

Let the greater part be x, then the less will be $7 - x$; so that $x = 7 - x + 3$, or $x = 10 - x$. Adding x, we have $2x = 10$; and dividing by 2, $x = 5$.

The two parts therefore are 5 and 2.

Question 2. It is required to divide a into two parts, so that the greater may exceed the less by b.

Let the greater part be x, then the other will be $a - x$; so that $x = a - x + b$. Adding x, we have $2x = a + b$; and dividing by 2, $x = \frac{a+b}{2}$.

Another method of solution. Let the greater part $= x$; which as it exceeds the less by b, it is evident that this is less than the other by b, and therefore must be $= x - b$. Now,