SECTION IV.

Of Algebraic Equations, *and of the* Resolution *of those* Equations.

CHAP. I.

Of the Solution *of* Problems *in general.*

563. The principal object of Algebra, as well as of all the other branches of Mathematics, is to determine the value of quantities that were before unknown; and this is obtained by considering attentively the conditions given, which are always expressed in known numbers. For this reason, Algebra has been defined, *The science which teaches how to determine unknown quantities by means of those that are known.*

564. The above definition agrees with all that has been hitherto laid down: for we have always seen that the knowledge of certain quantities leads to that of other quantities, which before might have been considered as unknown.

Of this, Addition will readily furnish an example; for, in order to find the sum of two or more given numbers, we had to seek for an unknown number, which should be equal to those known numbers taken together. In Subtraction, we sought for a number which should be equal to the difference of two known numbers. A multitude of other examples are presented by Multiplication, Division, the Involution of powers, and the Extraction of roots; the question being always reduced to finding, by means of known quantities, other quantities which are unknown.

565. In the last section, also, different questions were resolved, in which it was required to determine a number that could not be deduced from the knowledge of other given numbers, except under certain conditions. All those questions were reduced to finding, by the aid of some given numbers, a new number, which should have a certain connexion with them; and this connexion was determined by

certain conditions, or properties, which were to agree with the quantity sought.

566. In Algebra, when we have a question to resolve, we represent the number sought by one of the last letters of the alphabet, and then consider in what manner the given conditions can form an equality between two quantities. This equality is represented by a kind of formula, called an *equation*, which enables us finally to determine the value of the number sought, and consequently to resolve the question. Sometimes several numbers are sought; but they are found in the same manner by equations.

567. Let us endeavour to explain this farther by an example. Suppose the following question, or *problem*, was proposed:

Twenty persons, men and women, dine at a tavern; the share of the reckoning for one man is 8 shillings, for one woman 7 shillings, and the whole reckoning amounts to *7l. 5s.* Required the number of men and women separately?

In order to resolve this question, let us suppose that the number of men is $= x$; and, considering this number as known, we shall proceed in the same manner as if we wished to try whether it corresponded with the conditions of the question. Now, the number of men being $= x$, and the men and women making together twenty persons, it is easy to determine the number of the women, having only to subtract that of the men from 20, that is to say, the number of women must be $20 - x$.

But each man spends 8 shillings; therefore x men must spend $8x$ shillings. And since each woman spends 7 shillings, $20 - x$ women must spend $140 - 7x$ shillings. So that adding together $8x$ and $140 - 7x$, we see that the whole 20 persons must spend $140 + x$ shillings. Now, we know already how much they have spent; namely, *7l. 5s.* or *145s.*; there must be an equality, therefore, between $140 + x$ and 145; that is to say, we have the equation $140 + x = 145$, and thence we easily deduce $x = 5$, and consequently $20 - x = 20 - 5 = 15$; so that the company consisted of 5 men, and 15 women.

568. Again, Suppose twenty persons, men and women, go to a tavern; the men spend 24 shillings, and the women as much: but it is found that the men have spent 1 shilling each more than the women. Required the number of men and women separately?

Let the number of men be represented by x.

Then the women will be $20 - x$.

Now, the x men having spent 24 shillings, the share of each man is $\frac{24}{x}$. The $20 - x$ women having also spent 24 shillings, the share of each woman is $\frac{24}{20-x}$.

But we know that the share of each woman is one shilling less than that of each man; if, therefore, we subtract 1 from the share of a man, we must obtain that of a woman; and consequently $\frac{24}{x} - 1 = \frac{24}{20-x}$. This, therefore, is the equation, from which we are to deduce the value of x. This value is not found with the same ease as in the preceding question; but we shall afterwards see that $x = 8$, which value answers to the equation; for $\frac{24}{8} - 1 = \frac{24}{12}$ includes the equality $2 = 2$.

569. It is evident therefore how essential it is, in all problems, to consider the circumstances of the question attentively, in order to deduce from it an equation that shall express by letters the numbers sought, or unknown. After that, the whole art consists in resolving those equations, or deriving from them the values of the unknown numbers; and this shall be the subject of the present section.

570. We must remark, in the first place, the diversity which subsists among the questions themselves. In some, we seek only for one unknown quantity; in others, we have to find two, or more; and, it is to be observed, with regard to this last case, that, in order to determine them all, we must deduce from the circumstances, or the conditions of the problem, as many equations as there are unknown quantities.

571. It must have already been perceived, that an equation consists of two parts separated by the sign of equality, $=$, to shew that those two quantities are equal to one another; and we are often obliged to perform a great number of transformations on those two parts, in order to deduce from them the value of the unknown quantity: but these transformations must be all founded on the following principles, namely, That two equal quantities remain equal, whether we add to them, or subtract from them, equal quantities; whether we multiply them, or divide them, by the same number; whether we raise them both to the same power, or extract their roots of the same degree; or lastly,

whether we take the logarithms of those quantities, as we have already done in the preceding section.

572. The equations which are most easily resolved, are those in which the unknown quantity does not exceed the first power, after the terms of the equation have been properly arranged; and these are called *simple equations,* or *equations of the first degree.* But if, after having reduced an equation, we find in it the square, or the second power, of the unknown quantity, it is called an *equation of the second degree,* which is more difficult to resolve. *Equations of the third degree* are those which contain the cube of the unknown quantity, and so on. We shall treat of all these in the present section.

CHAP. II.

Of the Resolution *of* Simple Equations, *or* Equations *of the* First Degree.

573. When the number sought, or the unknown quantity, is represented by the letter x, and the equation we have obtained is such, that one side contains only that x, and the other simply a known number, as, for example, $x = 25$, the value of x is already known. We must always endeavour, therefore, to arrive at such a form, however complicated the equation may be when first obtained: and, in the course of this section, the rules shall be given, and explained, which serve to facilitate these reductions.

574. Let us begin with the simplest cases, and suppose, first, that we have arrived at the equation $x + 9 = 16$. Here we see immediately that $x = 7$: and, in general, if we have found $x + a = b$, where a and b express any known numbers, we have only to subtract a from both sides, to obtain the equation $x = b - a$, which indicates the value of x.

575 If we have the equation $x - a = b$, we must add a to both sides, and shall obtain the value of $x = b + a$. We must proceed in the same manner, if the equation have this form, $x - a = a^2 + 1$: for we shall find immediately $x = a^2 + a + 1$.

In the equation $x - 8a = 20 - 6a$, we find

$$x = 20 - 6a + 8a, \text{ or } x = 20 + 2a.$$