which gives 1 for the product, as the nature of the thing required.

377. If we multiply the series which we found for the value of $\frac{1}{(a+b)^2}$, by $a+b$ only, the product ought to answer to the fraction $\frac{1}{a+b}$, or be equal to the series already found, namely, $\frac{1}{a}-\frac{b}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^4}+\frac{b^4}{a^5}$, &c. and this the actual multiplication will confirm.

$$\frac{1}{a^2}-\frac{2b}{a^3}+\frac{3b^2}{a^4}-\frac{4b^3}{a^5}+\frac{5b^4}{a^6}, \text{ \&c.}$$
$$a+b$$

$$\frac{1}{a}-\frac{2b}{a^2}+\frac{3b^2}{a^3}-\frac{4b^3}{a^4}+\frac{5b^4}{a^5}, \text{ \&c.}$$
$$+\frac{b}{a^2}-\frac{2b^2}{a^3}+\frac{3b^3}{a^4}-\frac{4b^4}{a^5}, \text{ \&c.}$$

$$\frac{1}{a}-\frac{b}{a^2}+\frac{b^2}{a^3}-\frac{b^3}{a^4}+\frac{b^4}{a^5}, \text{ \&c. as required.}$$

SECTION III.

Of Ratios *and* Proportions.

CHAP. I.

Of Arithmetical Ratio, *or of the* Difference *between two* Numbers.

378. Two quantities are either equal to one another, or they are not. In the latter case, where one is greater than the other, we may consider their inequality under two different points of view: we may ask, *how much* one of the quantities is greater than the other? Or we may ask, *how many times* the one is greater than the other? The

results which constitute the answers to these two questions are both called *relations*, or *ratios*. We usually call the former an *arithmetical ratio*, and the latter a *geometrical ratio*, without however these denominations having any connexion with the subject itself. The adoption of these expressions has been entirely arbitrary.

379. It is evident, that the quantities of which we speak must be of one and the same kind; otherwise we could not determine any thing with regard to their equality, or inequality: for it would be absurd to ask if two pounds and three ells are equal quantities. So that in what follows, quantities of the same kind only are to be considered; and as they may always be expressed by numbers, it is of numbers only that we shall treat, as was mentioned at the beginning.

380. When of two given numbers, therefore, it is required how much the one is greater than the other, the answer to this question determines the arithmetical ratio of the two numbers; but since this answer consists in giving the difference of the two numbers, it follows, that an arithmetical ratio is nothing but the *difference* between two numbers; and as this appears to be a better expression, we shall reserve the words *ratio* and *relation* to express geometrical ratios.

381. As the difference between two numbers is found by subtracting the less from the greater, nothing can be easier than resolving the question how much one is greater than the other: so that when the numbers are equal, the difference being nothing, if it be required how much one of the numbers is greater than the other, we answer, by nothing; for example, 6 being equal to 2×3, the difference between 6 and 2×3 is 0.

382. But when the two numbers are not equal, as 5 and 3, and it is required how much 5 is greater than 3, the answer is, 2; which is obtained by subtracting 3 from 5. Likewise 15 is greater than 5 by 10; and 20 exceeds 8 by 12.

383. We have therefore three things to consider on this subject; 1st. the greater of the two numbers; 2d. the less; and 3d. the difference: and these three quantities are so connected together, that any two of the three being given, we may always determine the third.

Let the greater number be a, the less b, and the difference d; then d will be found by subtracting b from a, so that $d = a - b$; whence we see how to find d, when a and b are given.

384. But if the difference and the less of the two numbers, that is, if d and b were given, we might determine the greater number by adding together the difference and the less number, which gives $a = b + d$; for if we take from $b + d$ the less number b, there remains d, which is the known difference: suppose, for example, the less number is 12, and the difference 8, then the greater number will be 20.

385. Lastly, if beside the difference d, the greater number a be given, the other number b is found by subtracting the difference from the greater number, which gives $b = a - d$; for if the number $a - d$ be taken from the greater number a, there remains d, which is the given difference.

386. The connexion, therefore, among the numbers, a, b, d, is of such a nature as to give the three following results: 1st. $d = a - b$; 2d. $a = b + d$; 3d. $b = a - d$; and if one of these three comparisons be just, the others must necessarily be so also: therefore, generally, if $z = x + y$, it necessarily follows, that $y = z - x$, and $x = z - y$.

387. With regard to these arithmetical ratios we must remark, that if we add to the two numbers a and b, any number c, assumed at pleasure, or subtract it from them, the difference remains the same; that is, if d is the difference between a and b, that number d will also be the difference between $a + c$ and $b + c$, and between $a - c$ and $b - c$. Thus, for example, the difference between the numbers 20 and 12 being 8, that difference will remain the same, whatever number we add to, or subtract from, the numbers 20 and 12.

388. The proof of this is evident: for if $a - b = d$, we have also $(a + c) - (b + c) = d$; and likewise $(a - c) - (b - c) = d$.

389. And if we double the two numbers a and b, the difference will also become double; thus, when $a - b = d$, we shall have $2a - 2b = 2d$; and, generally, $na - nb = nd$, whatever value we give to n.