markable, that an infinite series, though it never ceases, may have a determinate value. It should likewise be observed, that, from this branch of mathematics, inventions of the utmost importance have been derived; on which account the subject deserves to be studied with the greatest attention.

QUESTIONS FOR PRACTICE.

1. Resolve $\frac{ax}{a-x}$ into an infinite series.

Ans. $x+\frac{x^2}{a}+\frac{x^3}{a^2}+\frac{x^4}{a^3}$, &c.

2. Resolve $\frac{b}{a+x}$ into an infinite series.

Ans. $\frac{b}{a}\times(1-\frac{x}{a}+\frac{x^2}{a^2}-\frac{x^3}{a^3}+$, &c.)

3. Resolve $\frac{a^2}{x+b}$ into an infinite series.

Ans. $\frac{a^2}{x}\times(1-\frac{b}{x}+\frac{b^2}{x^2}-\frac{b^3}{x^3}+$, &c.)

4. Resolve $\frac{1+x}{1-x}$ into an infinite series.

Ans. $1+2x+2x^2+2x^3+2x^4$, &c.

5. Resolve $\frac{a^2}{(a+x)^2}$ into an infinite series.

Ans. $1-\frac{2x}{a}+\frac{3x^2}{a^2}-\frac{4x^3}{a^3}$, &c.

CHAP. VI.

Of the Squares *of* Compound Quantities.

306. When it is required to find the square of a compound quantity, we have only to multiply it by itself, and the product will be the square required.

For example, the square of $a+b$ is found in the following manner:

$$\begin{array}{l} a+b \\ a+b \\ \hline a^2+ab \\ \quad\;\; ab+b^2 \\ \hline a^2+2ab+b^2 \\ \hline \end{array}$$

307. So that when the root consists of two terms added together, as $a + b$, the square comprehends, 1st, the squares of each term, namely, a^2 and b^2; and 2dly, twice the product of the two terms, namely, $2ab$: so that the sum $a^2 + 2ab + b^2$ is the square of $a + b$. Let, for example, $a = 10$, and $b = 3$; that is to say, let it be required to find the square of $10 + 3$, or 13, and we shall have $100 + 60 + 9$, or 169.

308. We may easily find, by means of this formula, the squares of numbers, however great, if we divide them into two parts. Thus, for example, the square of 57, if we consider that this number is the same as $50 + 7$, will be found $= 2500 + 700 + 49 = 3249$.

309. Hence it is evident, that the square of $a + 1$ will be $a^2 + 2a + 1$: and since the square of a is a^2, we find the square of $a + 1$ by adding to that square $2a + 1$; and it must be observed, that this $2a + 1$ is the sum of the two roots a and $a + 1$.

Thus, as the square of 10 is 100, that of 11 will be $100 + 21$: the square of 57 being 3249, that of 58 is $3249 + 115 = 3364$; the square of $59 = 3364 + 117 = 3481$; the square of $60 = 3481 + 119 = 3600$, &c.

310. The square of a compound quantity, as $a + b$, is represented in this manner $(a + b)^2$. We have therefore $(a + b)^2 = a^2 + 2ab + b^2$, whence we deduce the following equations:

$$(a+1)^2=a^2+2a+1; \qquad (a+2)^2=a^2+4a+4;$$
$$(a+3)^2=a^2+6a+9; \qquad (a+4)^2=a^2+8a+16; \text{ \&c.}$$

311. If the root be $a - b$, the square of it is $a^2 - 2ab + b^2$, which contains also the squares of the two terms, but in such a manner, that we must take from their sum twice the product of those two terms. Let, for example, $a = 10$, and $b = -1$, then the square of 9 will be found equal to $100 - 20 + 1 = 81$.

312. Since we have the equation $(a - b)^2 = a^2 - 2ab + b^2$, we shall have $(a - 1)^2 = a^2 - 2a + 1$. The square of $a - 1$ is found, therefore, by subtracting from a^2 the sum of the two roots a and $a - 1$, namely, $2a - 1$. Thus, for

example, if $a = 50$, we have $a^2 = 2500$, and $2a - 1 = 99$; therefore $49^2 = 2500 - 99 = 2401$.

313. What we have said here may be also confirmed and illustrated by fractions; for if we take as the root $\frac{3}{5} + \frac{2}{5} = 1$, the square will be, $\frac{9}{25} + \frac{4}{25} + \frac{12}{25} = \frac{25}{25} = 1$.

Farther, the square of $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ will be $\frac{1}{4} - \frac{1}{3} + \frac{1}{9} = \frac{1}{36}$.

314. When the root consists of a greater number of terms, the method of determining the square is the same. Let us find, for example, the square of $a + b + c$:

$$\begin{array}{l} a+b+c \\ a+b+c \\ \hline a^2+ab+ac \\ \quad\; ab+b^2+bc \\ \qquad\quad ac+bc+c^2 \\ \hline a^2+2ab+2ac+b^2+2bc+c^2 \end{array}$$

We see that it contains, first, the square of each term of the root, and beside that, the double products of those terms multiplied two by two.

315. To illustrate this by an example, let us divide the number 256 into three parts, $200 + 50 + 6$; its square will then be composed of the following parts:

$$\begin{array}{rcr} 200^2 & = & 40000 \\ 50^2 & = & 2500 \\ 6^2 & = & 36 \\ 2\,(50 \times 200) & = & 20000 \\ 2\,(\ 6 \times 200) & = & 2400 \\ 2\,(\ 6 \times \ 50) & = & 600 \\ \hline & & 65536 = 256 \times 256, \text{ or } 256^2. \end{array}$$

316. When some terms of the root are negative, the square is still found by the same rule; only we must be careful what signs we prefix to the double products. Thus, $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$; and if we represent the number 256 by $300 - 40 - 4$, we shall have,

Positive Parts.		Negative Parts.	
$300^2 =$	90000	$2(40 \times 300) =$	24000
$40^2 =$	1600	$2(\ 4 \times 300) =$	2400
$2(40 \times 4) =$	320		-26400
$4^2 =$	16		
	91936		
	-26400		
	65536, the square of 256 as before.		

CHAP. VII.

Of the Extraction *of* Roots *applied to* Compound Quantities.

317. In order to give a certain rule for this operation, we must consider attentively the square of the root $a+b$, which is $a^2+2ab+b^2$, in order that we may reciprocally find the root of a given square.

318. We must consider therefore, first, that as the square, $a^2+2ab+b^2$, is composed of several terms, it is certain that the root also will comprise more than one term; and that if we write the terms of the square in such a manner, that the powers of one of the letters, as a, may go on continually diminishing, the first term will be the square of the first term of the root; and since, in the present case, the first term of the square is a^2, it is certain that the first term of the root is a.

319. Having therefore found the first term of the root, that is to say, a, we must consider the rest of the square, namely, $2ab+b^2$, to see if we can derive from it the second part of the root, which is b. Now, this remainder, $2ab+b^2$, may be represented by the product, $(2a+b)b$; wherefore the remainder having two factors, $(2a+b)$, and b, it is evident that we shall find the latter, b, which is the second part of the root, by dividing the remainder, $2ab+b^2$, by $2a+b$.

320. So that the quotient, arising from the division of the above remainder by $2a+b$, is the second term of the root required; and in this division we observe, that $2a$ is the double of the first term a, which is already determined: so that although the second term is yet unknown, and it is necessary, for the present, to leave its place empty, we may nevertheless attempt the division, since in it we attend only