SECTION II.

Of the different Methods *of calculating* Compound Quantities.

CHAP. I.

Of the Addition *of* Compound Quantities.

256. When two or more expressions, consisting of several terms, are to be added together, the operation is frequently represented merely by signs, placing each expression between two parentheses, and connecting it with the rest by means of the sign $+$. Thus, for example, if it be required, to add the expressions $a + b + c$ and $d + e + f$, we represent the sum by

$$(a + b + c) + (d + e + f).$$

257. It is evident that this is not to perform addition, but only to represent it. We see, however, at the same time, that in order to perform it actually, we have only to leave out the parentheses; for as the number $d + e + f$ is to be added to $a + b + c$, we know that this is done by joining to it first $+ d$, then $+ e$, and then $+ f$; which therefore gives the sum $a + b + c + d + e + f$; and the same method is to be observed, if any of the terms are affected by the sign $-$; as they must be connected in the same way, by means of their proper sign.

258. To make this more evident, we shall consider an example in pure numbers, proposing to add the expression $15 - 6$ to $12 - 8$. Here, if we begin by adding 15, we shall have $12 - 8 + 15$; but this is adding too much, since we had only to add $15 - 6$, and it is evident that 6 is the number which we have added too much; let us therefore take this 6 away by writing it with the negative sign, and we shall have the true sum,

$$12 - 8 + 15 - 6;$$

which shews that the sums are found by writing all the terms, each with its proper sign.

259. If it were required therefore to add the expression $d - e - f$ to $a - b + c$, we should express the sum thus,

$$a - b + c + d - e - f;$$

remarking, however, that it is of no consequence in what order we write these terms; for their places may be changed at pleasure, provided their signs be preserved; so that this sum might have been written thus,

$$c - e + a - f + d - b.$$

260. It is evident, therefore, that addition is attended with no difficulty, whatever be the form of the terms to be added: thus, if it were necessary to add together the expressions $2a^3 + 6\sqrt{b} - 4\log. c$ and $5\sqrt[5]{a} - 7c$, we should write them

$$2a^3 + 6\sqrt{b} - 4\log. c + 5\sqrt[5]{a} - 7c,$$

either in this or in any other order of the terms; for if the signs are not changed, the sum will always be the same.

261. But it frequently happens that the sums represented in this manner may be considerably abridged, as is the case when two or more terms destroy each other; for example, if we find in the same sum the terms $+a - a$, or $3a - 4a + a$: or when two or more terms may be reduced to one, &c. Thus, in the following examples:

$$\begin{array}{ll} 3a + 2a = 5a, & 7b - 3b = +4b \\ -6c + 10c = +4c; & 4d - 2d = 2d \\ 5a - 8a = -3a, & -7b + b = -6b \\ -3c - 4c = -7c, & -3d - 5d = -8d \\ 2a - 5a + a = -2a, & -3b - 5b + 2b = -6b. \end{array}$$

Whenever two or more terms, therefore, are entirely the same with regard to letters, their sum may be abridged; but those cases must not be confounded with such as these, $2a^2 + 3a$, or $2b^3 - b^4$, which admit of no abridgment.

262. Let us consider now some other examples of reduction, as the following, which will lead us immediately to an important truth. Suppose it were required to add together the expressions $a + b$ and $a - b$; our rule gives $a + b + a - b$; now $a + a = 2a$, and $b - b = 0$; the sum therefore is $2a$: consequently, if we add together the sum of two numbers $(a + b)$ and their difference $(a - b)$, we obtain the double of the greater of those two numbers.

This will be better understood perhaps from the following examples:

$$\begin{array}{rr} 3a - 2b - c & a^3 - 2a^2b + 2ab^2 \\ 5b - 6c + a & -\ a^2b + 2ab^2 - b^3 \\ \hline 4a + 3b - 7c & a^3 - 3a^2b + 4ab^2 - b^3 \end{array}$$

$$\begin{array}{r}4a^2-3b+\ 2c\\3a^2+2b-12c\\\hline 7a^2-\ b+10c\\\hline\end{array}\qquad\begin{array}{r}a^4+2ab+\ b^3\\-a^4-2a^2b+3b^3\\\hline -2a^2b+2ab+4b^3\\\hline\end{array}$$

CHAP. II.

Of the Subtraction *of* Compound Quantities.

263. If we wish merely to represent subtraction, we enclose each expression within two parentheses, joining, by the sign —, the expression which is to be subtracted, to that from which we have to subtract it.

When we subtract, for example, the expression $d - e + f$ from the expression $a - b + c$, we write the remainder thus:

$$(a - b + c) - (d - e + f);$$

and this method of representing it sufficiently shews which of the two expressions is to be subtracted from the other.

264. But if we wish to perform the actual subtraction, we must observe, first, that when we subtract a positive quantity $+b$ from another quantity a, we obtain $a-b$: and secondly, when we subtract a negative quantity $-b$ from a, we obtain $a + b$; because to free a person from a debt is the same as to give him something.

265. Suppose now it were required to subtract the expression $b - d$ from $a - c$. We first take away b, which gives $a - c - b$: but this is taking away too much by the quantity d, since we had to subtract only $b - d$; we must therefore restore the value of d, and then shall have

$$a - c - b + d;$$

whence it is evident that the terms of the expression to be subtracted must change their signs, and then be joined, with those contrary signs, to the terms of the other expression.

266. Subtraction is therefore easily performed by this rule, since we have only to write the expression from which we are to subtract, joining the other to it without any change beside that of the signs. Thus, in the first example, where it was required to subtract the expression $d - e + f$ from $a - b + c$, we obtain $a - b + c - d + e - f$.

An example in numbers will render this still more clear;